

ELEVENTH LECTURE SUMMARY

ANALYSIS OF VARIANCE

TESTS WHETHER k POPULATIONS ALL HAVE THE **SAME MEAN** (THE **NULL** HYPOTHESIS) OR **NOT** (THE **ALTERNATE** HYPOTHESIS).

WE **ASSUME** (AT LEAST APPROXIMATELY) THAT THE POPULATIONS ARE **NORMAL**, AND THEIR STANDARD DEVIATIONS ARE **IDENTICAL**.

TO COMPUTE **TEST STATISTIC** IS NOW A LOT MORE ELABORATE. WE START BY COMPUTING

$$SS = \sum x^2 - \frac{(\sum x)^2}{n}$$

INDIVIDUALLY FOR EACH SAMPLE. THE **SUM** OF THESE IS CALLED **SS_w** (**WITHIN** OR **ERROR** SUM OF SQUARES).

THEN, WE POOL ALL k SAMPLES INTO ONE, AND FIND THE CORRESPONDING TOTAL SUM OF SQUARES

$$SS_{TOT} = \sum_{TOT} x^2 - \frac{(\sum_{TOT} x)^2}{N}$$

(N IS THE NUMBER OF ALL OBSERVATIONS)

FINALLY, $SS_{BET} = SS_{TOT} - SS_W$ OR EQUIVALENTLY:

$$SS_{BET} = \sum_{ALL\ SAMPLES} \frac{(\sum x)^2}{n} - \frac{(\sum_{TOT} x)^2}{N}$$

(THE BETWEEN OR FACTOR SUM OF SQUARES)

THE RESULTS ARE USUALLY SUMMARIZED IN THE FOLLOWING TABLE:

SOURCE	SS	D.F.	MS	F RATIO
BETWEEN	SS_{BET}	$k - 1$	$SS_{\text{BET}} / (k-1)$	$MS_{\text{BET}} / MS_{\text{W}}$
WITHIN	SS_{W}	$N - k$	$SS_{\text{W}} / (N-k)$	
TOTAL	SS_{TOT}	$N - 1$		

THE RESULTING TEST STATISTIC HAS, WHEN H_0 IS TRUE, THE SO CALLED **F** (FISHER) DISTRIBUTION.

ITS **CRITICAL VALUES** (THIS IS, ONE MORE TIME, ALWAYS A **RIGHT-TAIL** TEST) ARE FOUND IN TABLE 8.

THIS TIME, WE NEED **TWO DEGREES OF FREEDOM**, **NUMERATOR** ($k - 1$) AND **DENOMINATOR** ($N - k$).