FOURTH LECTURE SUMMARY

NORMAL DISTRIBUTION

IS THE MOST IMPORTANT OF ALL DISTRIBUTIONS. IT IS OF THE **CONTINUOUS** TYPE, WHICH MEANS WE CANNOT COMPUTE PROBABILITIES OF THE Pr(X = 3.76571) TYPE, THE ONLY MEANINGFUL QUESTIONS MUST INVOLVE A **RANGE** OF VALUES, E.G. Pr(3.2 < X < 4.7)

THE NORMAL DISTRIBUTION IS FULLY SPECIFIED BY ITS MEAN : AND STANDARD DEVIATION F (ITS PARAMETERS)

A VERY IMPORTANT SPECIAL CASE OF THIS DISTRIBUTION IS CALLED STANDARD (OR STANDARDIZED), DENOTED *Z* - IT IS THE ONLY CASE FOR WHICH WE HAVE A TABLE OF PROBABILITIES. LUCKILY, ANY OTHER NORMAL RANDOM VARIABLE *X* CAN BE CONVERTED TO THIS *Z* BY

$$Z = \frac{X - \mu}{\sigma}$$

TABLE 5 LISTS THE PROBABILITIES OF Pr(Z < z) FOR z = -3.49, -3.48, ... 0.00, 0.01, 0.02, ... 3.48, 3.49 (ANYTHING SMALLER, THE ANSWER IS 0, ANYTHING BIGGER, AND THE ANSWER IS 1)

BASED ON THIS, WE SHOULD BE ABLE TO COMPUTE THE PROBABILITY OF *Z* BEING INSIDE ANY (ONE OR TWO ENDS) RANGE, E.G. Z < 1.32 (N.B. *Z*#1.32 IS THE SAME QUESTION, THE EQUAL SIGNS CAN ALL BE IGNORED NOW), Z < -1.32, -2.06 < Z < 0.14, ETC.

SIMILARLY, WHEN *X* HAS ANY OTHER (NON- STANDARD) NORMAL DISTRIBUTION, WITH A GIVEN : AND F, WE FIRST REWRITE THE QUESTION (AND THE CORRESPONDING INEQUALITIES) IN TERMS OF *Z* = $\frac{X-\mu}{\sigma}$ AND GO BACK TO OUR TABLES.

WHEN Z HAS THE STANDARD NORMAL DISTRIBUTION, WE SHOULD ALSO BE ABLE TO ANSWER QUESTIONS LIKE:

FIND z SUCH THAT $\Pr(Z < z) = 95\%$ OR $\Pr(|Z| < z)$ / $\Pr(-z < Z < z) = 99\%$, ETC.

THIS IS DONE BY LOOKING UP THE RELEVANT (CLOSEST TO 0.9500, 0.9950 ETC.) PROBABILITY IN OUR NORMAL TABLES, AND READING OFF THE CORRESPONDING VALUE OF z (A GRAPH MAY HELP).

THE SAME KIND OF QUESTION (E.G. FIND x SUCH THAT $\Pr(X < x) = 90\%$) CAN BE ANSWERED FOR <u>ANY</u> NORMALLY DISTRIBUTED RANDOM VARIABLE X (GIVEN : AND F) BY:

1) FINDING z SUCH THAT $\Pr(Z < z) = 90\%$
2) CONVERTING THIS TO $x = z \times F + :$

NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTION

IS LEGITIMATE WHEN BOTH *np* AND *nq* ARE BIGGER THAN 5. WE CAN THEN TREAT *X* AS NORMAL, WITH : = np AND F = $\sqrt{npq}$ (DON'T FORGET THE CONTINUITY CORRECTION).

TO POISSON DISTRIBUTION

LEGITIMATE WHEN 8 > 30 (: = 8, F = $\sqrt{\lambda}$)