

# SIXTH LECTURE SUMMARY

## CONFIDENCE INTERVAL FOR :

SOLVING THE TWO INEQUALITIES IN

$$\Pr\left(-1.96 < \frac{\bar{x} - \mu}{s / \sqrt{n}} < 1.96\right) = 95\%$$

FOR : (ASSUMING THAT  $\bar{x}$  AND  $s$  HAVE ALREADY BEEN COMPUTED), WE GET:

$$\bar{x} - 1.96 \times \frac{s}{\sqrt{n}} < \mu < \bar{x} + 1.96 \times \frac{s}{\sqrt{n}}$$

CALLED THE 95% CONFIDENCE INTERVAL FOR THE UNKNOWN VALUE OF : .

$\bar{x}$  IS THE POINT ESTIMATE OF :

$1.96 \times \frac{s}{\sqrt{n}}$  / E IS THE MAXIMAL ERROR

TOLERANCE (MARGIN OF ERROR, ALSO EFFECTIVELY THE ACCURACY, THE  $\pm$ ) OF THE FORMULA

IN **GENERAL**, THE TWO LIMITS ARE COMPUTED BY:

$$\bar{x} \pm z_c \times \frac{s}{\sqrt{n}}$$

WHERE  $z_c$  IS THE CORRESPONDING **CRITICAL VALUE** (**TABLE 5 b** - IF WE CAN FIND IT THERE), DETERMINED BASED ON THE **LEVEL OF CONFIDENCE** (CHOSEN BY US)

REPLACE  $z_c$  BY  $t_c$  (**TABLE 6 OR INSERT**) WHENEVER  $n < 30$  AND THE POPULATION IS **NORMAL** (d.f. =  $n - 1$ )

REPLACE  $s$  BY  $F$  WHENEVER THE LATTER IS AVAILABLE (THEN, USE  $z_c$  FOR ANY  $n$ )

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TO ESTIMATE HOW LARGE A SAMPLE WE

NEED, USE: 
$$n = \left( \frac{z_c \times s}{E} \right)^2$$

WHERE  $E$  IS THE DESIRED **MAXIMAL ERROR TOLERANCE** (THE ACCURACY WE WANT TO ACHIEVE).

NOTE THAT AN  $s$  (PREVIOUS SMALL SAMPLE), OR A KNOWN  $F$  (IN PLACE OF  $s$ ) ARE NEEDED.

## CONFIDENCE INTERVAL FOR $p$ (POPULATION PROPORTION)

$$\hat{p} \pm z_c \times \sqrt{\frac{\hat{p} \times \hat{q}}{n}}$$

WHERE  $\hat{p}$  IS THE SAMPLE PROPORTION OF 'SUCCESSSES',  $\hat{q} = 1 - \hat{p}$ , AND  $n$  MUST BE 'LARGE' (AT LEAST 30).

TO ESTIMATE HOW LARGE AN  $n$  WE NEED, USE:

$$\hat{p} \times \hat{q} \times \left( \frac{z_c}{E} \right)^2$$

IF A PREVIOUS (SMALL SAMPLE) ESTIMATE OF  $\hat{p}$  IS AVAILABLE,

OTHERWISE, USE

$$n = \frac{1}{4} \times \left( \frac{z_c}{E} \right)^2$$

(THE 'WORST CASE' SCENARIO)

THE ANSWER MUST BE **AN INTEGER**  
(ROUND UPWARDS, TO BE ON A SAFE  
SIDE)