Set 4.1

Question 8:

$$c_{i+2} = \frac{4c_i}{(i+2)(i+1)}$$

generates

$$\begin{array}{cc} c_0 = 1 & c_0 = 0 \\ c_1 = 0 & c_1 = 1 \\ c_2 = \frac{4}{2!} & c_2 = 0 \\ c_3 = 0 & c_3 = \frac{4}{3!} \\ c_4 = \frac{4^2}{4!} & c_4 = 0 \\ c_5 = 0 & c_5 = \frac{4^2}{5!} \\ c_6 = \frac{4^3}{6!} & c_6 = 0 \\ ... & ... \end{array}$$

The first sequence corresponds to $y_1 = \cosh(2x)$, the second one yields $y_2 = \frac{\sinh(2x)}{2}$. General solution:

$$y = A\cosh(2x) + B\sinh(2x)$$

Question 10:

$$c_{i+1} = \frac{ic_i}{(i+1)i} = \frac{c_i}{(i+1)} \qquad i = 2, 3, 4, ...$$

generates

$$\begin{array}{cc} c_0 = 1 & c_0 = 0 \\ c_1 = 0 & c_1 = 1 \\ c_2 = 0 & c_2 = \frac{1}{2} \\ c_3 = 0 & c_3 = \frac{1}{3!} \\ c_4 = 0 & c_4 = \frac{1}{4!} \\ c_5 = 0 & c_5 = \frac{1}{5!} \\ c_6 = 0 & c_6 = \frac{1}{6!} \\ ... & ... \end{array}$$

The first sequence yields $y_1 = 1$, the second one $y_2 = e^x - 1$. General solution:

$$y = A + Be^x$$

Set 4.2

Question 6:

$$\begin{aligned}
&\sum_2 i(i-1)c_i x^{i-2} - 4\sum_0 ic_i x^i + 4\sum_0 c_i x^{i+2} - 2\sum_0 c_i x^i \\
=\ & 2c_2 - 2c_0 + (6c_3 - 6c_1)x \\
&+ \sum_2 \left[(i+2)(i+1)c_{i+2} - (4i+2)c_i + 4c_{i-2}\right] x^i
\end{aligned}$$

generates

$$\begin{array}{cc}
c_0 = 1 & c_0 = 0 \\
c_1 = 0 & c_1 = 1 \\
c_2 = 1 & c_2 = 0 \\
c_3 = 0 & c_3 = 1 \\
c_4 = \frac{1}{2!} & c_4 = 0 \\
c_5 = 0 & c_5 = \frac{1}{2!} \\
c_6 = \frac{1}{3!} & c_6 = 0 \\
c_7 = 0 & c_7 = \frac{1}{3!} \\
c_8 = \frac{1}{4!} & c_8 = 0 \\
\ldots & \ldots
\end{array}$$

The first sequence yields $y_1 = \exp(x^2)$, the second one $y_2 = x\exp(x^2)$. General solution:

$$y = (A + Bx)\exp(x^2)$$

My question:

$$\begin{aligned}
&2\sum_2 i(i-1)c_i x^{i-2} + \sum_1 i(i-1)c_i x^{i-1} - \sum_0 ic_i x^i - \sum_0 c_i x^i \\
=\ & \sum_0 \left[2(i+2)(i+1)c_{i+2} + (i+1)ic_{i+1} - (i+1)c_i\right] x^i
\end{aligned}$$

implying

$$c_{i+2} = -\frac{ic_{i+1} - c_i}{2(i+2)}$$

generates

$$c_0 = 1$$
$$c_1 = -\tfrac{1}{2}$$
$$c_2 = \tfrac{1}{4}$$
$$c_3 = -\tfrac{1}{8}$$
$$c_4 = \tfrac{1}{16}$$
$$c_5 = -\tfrac{1}{2^5}$$
$$c_6 = \tfrac{1}{2^6}$$
$$c_7 = -\tfrac{1}{2^7}$$
$$c_8 = \tfrac{1}{2^8}$$
$$\ldots$$

This yields the following solution:

$$y = \frac{1}{1 + \frac{x}{2}} = \frac{2}{2 + x}$$