BROCK UNIVERSITY

Final Examination: April 2005
Course: MATH 2F05
Date of Examination: Apr. 15, 2005
Time of Examination: 9:00 - 12:00

Number of Pages: 2
Number of students: 16
Number of Hours: 3
Instructor: J. Vrbik

This is an open-book exam. Full credit given for **7** correct and complete answers.

In questions 1 to 4, solve the corresponding differential equation(s):

1.
$$(1-x^2)y'' + (x^2-2x-5)y' + 2(x+2)y = 0$$
subject to $y(0)=1$ and $y'(0)=2$, by the power-series technique.

2.
$$\begin{aligned} y_1' &= 3y_1 + 3y_2 + 2y_3 \\ y_2' &= -y_1 - 3y_2 + 3y_3 \\ y_3' &= -y_1 + 4y_2 - 4y_3 \end{aligned}$$
subject to $y_1(0) = \frac{3}{2}$, $y_2(0) = -1$, $y_3(0) = -1$.

3.
$$x^4 y^{iv} + 8x^3 y''' + x^2 y'' - 49xy' - 32y = x^2 \ln x$$
(particular solution should be found by the method of undetermined coefficients).

4.
$$4x^2 y'' + 12xy' + (4x-5)y = 0$$
by first introducing a new dependent variable $u \equiv x \cdot y$, and then a new independent variable $z \equiv 2 \cdot \sqrt{x}$ (show the details).

5. Verify the Stokes' theorem, using the following line integral:
$$\int_C (x^2,\ xy,\ 3) \bullet d\mathbf{r}$$
where C is a circle of radius 3, centered on $[-1,\ 2,\ -3]$ and parallel to the x-y plane (choose your own orientation).

6. Verify the Gauss' theorem, using the following surface integral:
$$\iint_S (x^2,\ xy,\ 3) \bullet d\mathbf{A}$$
where S is the (closed) surface of $\left\{ \begin{array}{c} x^2+y^2+z^2 \le 4 \\ 0 \le z \end{array} \right.$, oriented outwards.

Evaluate:

7.
$$\int_C \frac{z^2\,dz}{|z-i|^3}$$

where C is a straight-line segment from $2-3i$ to $-2+3i$

8.
$$\oint_C \frac{z^2 dz}{z^4 + i\ z^2 + 2}$$

where C is a circle of radius 1, centered on $\frac{i}{2}$ and oriented *clockwise*.

9.
$$\int_0^\infty \frac{x^2 dx}{x^4 + x^2 + 3}$$

by contour integration.

Find:

10. Curvature and torsion of the curve defined by $\left\{ \begin{array}{c} x^2 + 9y^2 = 9 \\ z = 1 + y^2 \end{array} \right.$, at $[3,\ 0,\ 1]$.

Hint: Parametrize the curve first.

11. Moment of inertia, with respect to the z axis, of the following 3-dimensional region:

$$x^2 + y^2 \le z \le 4$$

Assume uniform mass density, with the total mass of M.

12. Center of mass of the following surface: $\left\{ \begin{array}{c} x^2 + y^2 = z \\ z \le 4 \end{array} \right.$. Assume uniform mass density.