BROCK UNIVERSITY

Final Examination: April 1998	Number of Pages: 2
Course: MATH2F95	Number of students: 8
Date of Examination: Apr. 13, 1998	Number of Hours: 3
Time of Examination: 9:00-12:00	Instructor: J. Vrbik

This is an open-book exam. Full credit given for 8 correct and complete answers.

Solve:

1.
$$x^3y''' - 2x^2y'' - 3xy' + 3y = x^3$$
Hint: $y = x$ solves the corresponding homogeneous equation.

2.
$$\begin{aligned} \dot{x} &= 3x + 2y + 2t \\ \dot{y} &= -2x - y + 3 \end{aligned}$$

3.
$$xy'' + \frac{x-1}{x+1}y' + \frac{x-1}{(x+1)^2}y = 0$$

4.
$$x^2y'' + 3xy' + (x - \tfrac{5}{4})\, y = 0$$

Evaluate:

5.
$$\int_C y\,dx + z\,dy + x\,dz$$
where C: $\begin{cases} x^2 + y^2 = 1 \\ x + y + z = 1 \end{cases}$ oriented clockwise when viewed from above.

6.
$$\int_{(0,0,1)}^{(1,-1,2)} \frac{e^{x+y}}{z}dx + \left(\frac{e^{x+y}}{z} - ze^y\right)dy - \left(\frac{e^{x+y}}{z^2} + e^y + e^z\right)dz$$

7.
$$\iint_S (xy, yz, xz) \bullet d\mathbf{A}$$

where S is defined by $x^2 + y^2 + z^2 = a^2$ (oriented outwards).

8.
$$\int_C \frac{dz}{z-3}$$

where C is the straight-line segment from $-i$ to i.

9.
$$\oint_C \frac{z\,dz}{(z^2+z+1)^2}$$

where C is a clockwise circle of radius 2, centered at the origin.

10.
$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+x+1)^2}$$

Find:

11. The moment of inertia of a cylindrical shell (open top and bottom) of radius a and height h with respect to a line (axis of rotation) passing through its center and *perpendicular* to its axis of symmetry (assume that the total mass M is distributed uniformly over the cylinder's sides).

12. Redo the previous question assuming that the cylinder is solid (the mass is now distributed, uniformly, over its volume).