Progress Examination: December 2003	Number of Pages: 2
Course: MATH2F05	Number of students: 13
Date of Examination: Dec. 16, 2003	Number of Hours: 3
Time of Examination: 19:00 -22:00	Instructor: J. Vrbik

This is an open-book exam.
Full credit given for 7 complete answers (4 marks each).
Maple usage restricted to basic commands only.

1. Using the power-series technique, solve the following initial-value problem:
$$(1-2x)\,y'' + (2x-5)\,y' + 2\,y = 0$$
where $y(0) = 1$ and $y'(0) = 2$.

   A bonus mark will be given for finding the second basic solution.

2. Using the method of Frobenius, find a non-zero solution to
$$x^2(1+3x)\,y'' + x(5+21x)y' + (4+27x)y = 0$$
A bonus mark given for the second basic solution.

3. Find the family of curves orthogonal to
$$y = \exp(cx^2)$$
Draw a plot of several curves from each family.

Find the general solution to:

4.
$$\begin{aligned} y_1' &= y_1 + 2\,y_2 - 4e^x \\ y_2' &= -\,y_1 + 3\,y_2 - 3e^x \end{aligned}$$

5.
$$x^2y'' - 3xy' + (3-9x^4)y = 0$$
Express the solution in terms of Bessel functions!

6.

$$\mathbf{y}' = \begin{bmatrix} 14 & 18 & -2 & 18 \\ 0 & -1 & -5 & -5 \\ 10 & 8 & -8 & 8 \\ -10 & -13 & 7 & -9 \end{bmatrix} \mathbf{y}$$

The characteristic polynomial has two double roots.

7.

$$xy' + \exp(\frac{x}{y})y^3y' = y$$

Hint: $x \leftrightarrow y$.

8.

$$\begin{aligned} y_1' &= y_1 + y_2 + 3y_3 + 1 \\ y_2' &= -10y_1 + 8y_2 + 6y_3 - 2 \\ y_3' &= -5y_1 + y_2 + 9y_3 - 1 \end{aligned}$$

Anticipate a triple root of the characteristic polynomial. To find a particular solution, use undetermined coefficients!

9.

$$x^3y''' - 2x^2y'' + 3xy' - 2y = x^3 \ln x$$

10.

$$\begin{aligned} y + 2z + 3u &= -2 \\ x - y - 3z + u &= 7 \\ x + 2y + 3z + 10u &= 1 \\ x + 3y + 5z + 13u &= -1 \\ x + y + z + 7u &= 3 \end{aligned}$$

Express the solution in vector form. How would you describe the solution geometrically (a point, straight line, plane, ..)?

11.

$$y(\sin x + x\cos x)dx + (y^2\cos y - y^3\sin y - x\sin x)dy = 0$$