MATH 2F05 SECOND MIDTERM MARCH 2, 2005
Full credit given for three correct and complete answers.
Numerical answers should be given to 4 significant digits.
Open-book exam. Duration: 50 minutes

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1. Consider the following segment of a parabola:

$$y = x^2 \qquad \text{where} \quad 0 < x < 1$$

Find the length of this segment, and its center of mass. Also, the curvature at $x = \frac{1}{2}$. Hint: Parametrize the curve by $t \equiv x$.

2. Consider a two-dimensional region defined by

$$0 < y < x^2 \quad \text{where} \quad 0 < x < 1$$

Find the total area of this region, its center of mass, and its moment of inertia with respect to the $y = x$ axis (assume *unit* mass density).

3. Consider the following differential equation

$$4xy'' - 12y' + 9y = 0$$

(a) Introduce a new *dependent* variable $u(x)$ by

$$y = x^2 u$$

(spell out the new equation, in terms of u and x).

(b) Introduce a new *independent* variable $z = 3\sqrt{x}$. Indentify the resulting differential equation (with u as the dependent variable and z as the independent one).

(c) Use the result of a) and b) to find the general solution of the original equation.

4. Using the technique of Frobenius, find the *first* basic solution to

$$4x^2y'' - 8x^2y' + (1 + 4x^2)y = 0$$

(extra marks given for the second basic solution, when found either by the same technique, or by 'reduction of order' a.k.a. V of P).

5. Evaluate

$$\int_C [xy - z^2,\ xz - y^2,\ yz - x^2] \bullet d\mathbf{r}$$

where C consists of the following two (joint) segments: $[3, -1, 4] \rightarrow [2, 0, -6] \rightarrow [-3, 2, 1]$ (arrows implying straight-line connection).