

MATH2F95 SECOND MIDTERM FEBRUARY 27, 1998

Full credit given for three correct and complete answers.

Open-book exam.

Duration: 50 minutes

1. Solve $(x^2 + x - 2)y'' + (x + 1)y' - y = 0$.
2. Establish the shortest distance between the following two straight lines:

$$\begin{cases} 3x + 2y - 4z = 7 \\ 2x - 3y + z = 2 \end{cases} \quad \text{and} \quad \begin{cases} x - 4y + 3z = 1 \\ 4x + y - 2z = 5 \end{cases} .$$

3. Find the length of the first two coils (i.e. $0 < t < 4\pi$) of the following spiral:

$$\mathbf{r}(t) = [e^t \cos t, e^t \sin t, 0].$$

Also find the corresponding curvature and torsion at the point $[-e^\pi, 0, 0]$.

4. Evaluate:

$$\int_{(0,0,1)}^{(2,2,0)} \frac{e^x}{y+z} dx + \left(\frac{z}{(1-y)^2} - \frac{e^x}{(y+z)^2} \right) dy + \left(3z^2 + \frac{1}{1-y} - \frac{e^x}{(y+z)^2} \right) dz$$

5. Compute the volume of the following three-dimensional region (a cone with a spherical 'lid'):

$$\begin{cases} x^2 + y^2 + z^2 < 1 \\ z > \sqrt{x^2 + y^2} \end{cases}$$