Set 1.3:

4 Separable.

$$\begin{aligned} \frac{dy}{y^2} &= -3x^2 dx \\ -\frac{1}{y} &= -x^3 - C \\ y &= \frac{1}{C + x^3} \end{aligned}$$

8 Scale independent.

$$\begin{aligned} u + xu' &= 1 + u \\ du &= \frac{dx}{x} \\ u &= \ln|x| + C \\ y &= x(\ln|x| + C) \end{aligned}$$

10 Suggested substitution.

$$\begin{aligned} v' - 4 &= v^2 \\ \frac{dv}{v^2 + 4} &= dx \\ \frac{1}{2}\arctan\frac{v}{2} &= x + C \\ v &= 2\tan(2x + C) \\ y &= 2\tan(2x + C) - 4x \end{aligned}$$

16 Separable.

$$\begin{aligned} \frac{dy}{1 + 4y^2} &= dx \\ \frac{1}{2}\arctan(2y) &= x + C \\ y &= \frac{1}{2}\tan(2x + 2C) \\ y &= \frac{1}{2}\tan(2x) \end{aligned}$$

20 Modified scale independent.

$$\begin{aligned} u + xu' &= (u-1)^3x^2 + u \\ \frac{du}{(u-1)^3} &= x \cdot dx \\ -\frac{1}{2(u-1)^2} &= \frac{x^2}{2} + C \\ u &= \frac{1}{\sqrt{-2C - x^2}} + 1 \\ y &= \frac{x}{\sqrt{-2C - x^2}} + x \\ \frac{3}{2} &= \frac{1}{\sqrt{-2C - 1}} + 1 \\ -2C &= 5 \\ y &= \frac{x}{\sqrt{5 - x^2}} + x \end{aligned}$$

22 Modified scale independent.

$$\begin{aligned} u + xu' &= u + x\sec u \\ \cos u \cdot du &= dx \\ \sin u &= x + C \\ u &= \pi - \arcsin(x + C) \\ y &= x \cdot [\pi - \arcsin(x + C)] \\ \pi &= \pi - \arcsin(1 + C) \\ C &= -1 \\ y &= x \cdot [\pi - \arcsin(x - 1)] \end{aligned}$$

Set 1.6:

12 Linear. First remove RHS:

$$\begin{aligned} \frac{dy}{y} &= -3\frac{dx}{x} \\ \ln|y| &= -3\ln|x| + \ln c \\ y &= \frac{c}{x^3} \end{aligned}$$

Substitute:

$$\begin{aligned} x^3\left(\frac{c'}{x^3} - 3\frac{c}{x^4}\right) + 3x^3\frac{c}{x^3} &= \frac{1}{x} \\ c' &= \frac{1}{x} \\ c &= \ln|x| + C \\ y &= \frac{\ln|x| + C}{x^3} \end{aligned}$$

14 Linear. Remove RHS:

$$\begin{aligned} \frac{dy}{y} &= -2\frac{dx}{x} \\ \ln|y| &= -2\ln|x| + \ln c \\ y &= \frac{c}{x^2} \end{aligned}$$

Substitute:

$$\begin{aligned} x^2\left(\frac{c'}{x^2} - 2\frac{c}{x^3}\right) + 2x\frac{c}{x^2} &= \sinh(5x) \\ c' &= \sinh(5x) \\ c &= \frac{1}{5}\cosh(5x) + \tilde{C} \\ y &= \frac{\cosh(5x) + C}{5x^2} \end{aligned}$$

16 Linear. Remove RHS:

$$\begin{aligned} \frac{dy}{y} &= (1 + \frac{3}{x})dx \\ \ln|y| &= x + 3\ln|x| + \ln c \\ y &= cx^3e^x \end{aligned}$$

Substitute:

$$\begin{aligned} c'x^3e^x + 3cx^2e^x + cx^3e^x - (1 + \frac{3}{x})cx^3e^x &= x + 2 \\ c'x^3e^x &= x + 2 \\ c' &= (\frac{1}{x^2} + \frac{2}{x^3})e^{-x} \\ c &= -\frac{e^{-x}}{x^2} + C \\ y &= Cx^3e^x - x \\ e - 1 &= Ce - 1 \\ y &= x(x^2e^x - 1) \end{aligned}$$

22 Linear. Remove RHS:

$$\begin{aligned} \frac{dy}{y} &= -4\frac{dx}{x} \\ \ln|y| &= -4\ln|x| + \ln c \\ y &= \frac{c}{x^4} \end{aligned}$$

Substitute:

$$\begin{aligned} x\left(\frac{c'}{x^4} - 4\frac{c}{x^5}\right) + 4\frac{c}{x^4} &= 8x^4 \\ c' &= 8x^7 \\ c &= x^8 + C \\ y &= x^4 + \frac{C}{x^4} \\ 2 &= 1 + C \\ y &= x^4 + \frac{1}{x^4} \end{aligned}$$