

Set 1.6:

32 Bernoulli, $a = -1$, $1 - a = 2$

$$\begin{aligned}u' + 2u &= -2x \\ \frac{du}{u} &= -2dx \\ u &= c(x) \cdot e^{-2x} \\ c'e^{-2x} &= -2x \\ c &= -2 \int xe^{2x} dx = \\ & \quad \left(\frac{1}{2} - x\right)e^{2x} + C \\ u &= \frac{1}{2} - x + Ce^{-2x} \\ y &= \pm \sqrt{\frac{1}{2} - x + Ce^{-2x}}\end{aligned}$$

33 Bernoulli, $a = 4$, $1 - a = -3$

$$\begin{aligned}u' - u &= 2x - 1 \\ \frac{du}{u} &= dx \\ \ln u &= x + \ln c \\ y &= c(x)e^x \\ c'e^x + ce^x - ce^x &= 2x - 1 \\ c' &= (2x - 1)e^{-x} \\ c &= -(2x + 1)e^{-x} + C \\ u &= -(2x + 1) + Ce^x \\ y &= \frac{1}{\sqrt[3]{Ce^x - 2x - 1}}\end{aligned}$$

38 Bernoulli, $a = -1$, $1 - a = 2$

$$\begin{aligned}2xu' + 2(x-1)u &= 2x^2e^x \\ \frac{du}{u} &= \left(\frac{1}{x} - 1\right) dx \\ u &= c(x) \cdot xe^{-x} \\ x^2e^{-x}c' &= x^2e^x \\ c &= \int e^{2x} dx = \\ &= \frac{1}{2}e^{2x} + C \\ u &= \frac{x}{2}e^x + Cxe^{-x} \\ y &= \pm\sqrt{\frac{x}{2}e^x + Cxe^{-x}}\end{aligned}$$

Set 1.5:

12

$$\begin{aligned}\frac{d \cot y}{dy} &= -\csc^2 y \checkmark \\ G &= x \cot y + \frac{x^3}{3} \\ H &= -x \csc^2 y + x \csc^2 y = 0 \\ x \cot y + \frac{x^3}{3} &= \tilde{C} \\ \cot y &= \frac{C - x^3}{3x} \\ y &= \operatorname{arccot} \frac{C - x^3}{3x} = \arctan \frac{3x}{C - x^3}\end{aligned}$$

20

$$\begin{aligned}2x \exp(x^2) &= 2x \exp(x^2) \quad \text{YES, exact} \\ G &= \int 2xy \exp(x^2) dx = y \exp(x^2) \\ H &= \exp(x^2) - \exp(x^2) \\ y \exp(x^2) &= C \\ y &= C \exp(-x^2) \\ y &= 2 \exp(-x^2)\end{aligned}$$

28

$$\begin{aligned}\frac{6(y+1)}{x^4} &= 6\frac{y+1}{x^4} \quad \checkmark \\ G &= 3(y+1)^2 \int \frac{dx}{x^4} = -\frac{(y+1)^2}{x^3} \\ H &= -2\frac{(y+1)}{x^3} + 2\frac{(y+1)}{x^3} = 0 \\ (y+1)^2 &= \tilde{C}x^3 \\ y &= -1 + Cx^{3/2}\end{aligned}$$

34

$$\begin{aligned}\frac{-2\sin y - 0}{-\sin y} &= 2 \\ \ln F &= 2x \\ F &= e^{2x} \\ 2e^{2x} \cos y \, dx - e^{2x} \sin y \, dy &= 0 \\ G &= e^{2x} \cos y \\ H &= -e^{2x} \sin y + e^{2x} \sin y = 0 \\ e^{2x} \cos y &= C \\ y &= \arccos(Ce^{-2x})\end{aligned}$$