Set 2.9:

2

$$\begin{aligned}
\lambda^2 - 1 &= 0 \\
\lambda_{1,2} &= \pm 1 \\
y_{p_1} &= Axe^x \\
2Ae^x &= 2e^x \\
A &= 1 \\
y_{p_2} &= Be^{2x} \\
4Be^{2x} - Be^{2x} &= 6e^{2x} \\
B &= 2 \\
y &= (C_1 + x)e^x + C_2 e^{-x} + 2e^{2x}
\end{aligned}$$

6

$$\begin{aligned}
3\lambda^2 + 10\lambda + 3 &= 0 \\
\lambda_{1,2} &= -3, -\frac{1}{3} \\
y_{p_1} &= Ax + B \\
10A + 3Ax + 3B &= 9x \\
A &= 3,\ B = -10 \\
y_{p_2} &= C\sin x + D\cos x \\
& -3(C\sin x + D\cos x) + \\
& 10(C\cos x - D\sin x) + \\
& 3(C\sin x + D\cos x) = 5\cos x \\
D &= 0,\ C = \frac{1}{2} \\
y &= C_1 e^{-3x} + C_2 e^{-x/3} + 3x - 10 + \frac{1}{2}\sin x
\end{aligned}$$

8

$$\begin{aligned}
\lambda^2 + 6\lambda + 9 &= 0 \\
\lambda_{1,2} &= -3, -3 \\
y_p &= e^{-x}(A\sin x + B\cos x) \\
y_p' &= -e^{-x}(A\sin x + B\cos x) + \\
& e^{-x}(A\cos x - B\sin x) \\
y_p'' &= e^{-x}(A\sin x + B\cos x) \\
& -2e^{-x}(A\cos x - B\sin x) \\
& -e^{-x}(A\sin x + B\cos x)
\end{aligned}$$

Substitute:

$$\begin{aligned}
& -2e^{-x}(A\cos x - B\sin x) - 6e^{-x}(A\sin x + B\cos x) \\
& +6e^{-x}(A\cos x - B\sin x) + 9e^{-x}(A\sin x + B\cos x) = 50e^{-x}\cos x \\
& \begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} \\
& \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \end{bmatrix} \div (-25) = \begin{bmatrix} 8 \\ 6 \end{bmatrix} \\
y \quad = \quad & C_1 e^{-3x} + C_2 x e^{-3x} + (8\sin x + 6\cos x)e^{-x}
\end{aligned}$$

14

$$\begin{aligned}
\lambda^2 - 4\lambda + 20 &= 0 \\
\lambda_{1,2} &= 2 \pm 4i \\
y_p &= (A\sin x + B\cos x) \\
y_p' &= (A\cos x - B\sin x) \\
y_p'' &= -(A\sin x + B\cos x)
\end{aligned}$$

Substitute:

$$(-A + 4B + 20A)\sin x + (-B - 4A + 20B)\cos x = 377\sin x$$

$$\begin{aligned}
& \begin{bmatrix} 19 & 4 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 377 \\ 0 \end{bmatrix} \\
& \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 19 & -4 \\ 4 & 19 \end{bmatrix} \begin{bmatrix} 377 \\ 0 \end{bmatrix} \div 377 = \begin{bmatrix} 19 \\ 4 \end{bmatrix} \\
y \quad = \quad & e^{2x}\left[C_1 \sin(4x) + C_2 \cos(4x)\right] + 19\sin x + 4\cos x
\end{aligned}$$

Set 2.10:

4

$$\begin{aligned}
\lambda_{1,2} &= \pm 3i \\
u' &= \frac{\begin{vmatrix} 0 & \cos 3x \\ \frac{1}{\sin 3x} & -3\sin 3x \end{vmatrix}}{\begin{vmatrix} \sin 3x & \cos 3x \\ 3\cos 3x & -3\sin 3x \end{vmatrix}} = \frac{\cos 3x}{3\sin 3x} \\
u &= \frac{1}{9}\ln(\sin 3x) \\
v' &= \frac{\begin{vmatrix} \sin 3x & 0 \\ 3\cos x & \frac{1}{\sin 3x} \end{vmatrix}}{\begin{vmatrix} \sin 3x & \cos 3x \\ 3\cos 3x & -3\sin 3x \end{vmatrix}} = -\frac{1}{3} \\
v &= -\frac{x}{3} \\
y &= \left[C_1 + \frac{1}{9}\ln(\sin(3x))\right]\sin 3x + \left[C_2 - \frac{x}{3}\right]\cos 3x
\end{aligned}$$

8

$$\begin{aligned}
\lambda_{1,2} &= -3, -3 \\
u' &= \frac{\begin{vmatrix} 0 & xe^{-3x} \\ 16\frac{e^{-3x}}{x^2+1} & (-3x+1)e^{-3x} \end{vmatrix}}{\begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & (-3x+1)e^{-3x} \end{vmatrix}} = -16\frac{x}{x^2+1} \\
u &= -8\ln(1+x^2) \\
v' &= \frac{\begin{vmatrix} e^{-3x} & 0 \\ -3e^{-3x} & 16\frac{e^{-3x}}{x^2+1} \end{vmatrix}}{\begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & (-3x+1)e^{-3x} \end{vmatrix}} = \frac{16}{x^2+1} \\
v &= 16\arctan x \\
y &= \left[C_1 - 8\ln(1+x^2)\right]e^{-3x} + \left[C_2 + 16\arctan x\right]xe^{-3x}
\end{aligned}$$