Set 2.6:

2

$$\begin{aligned} m^2 - 5m + 6 &= 0 \\ m_{1,2} &= 2, 3 \\ y &= C_1 x^2 + C_2 x^3 \end{aligned}$$

6

$$\begin{aligned} m^2 - 2m + 2 &= 0 \\ m_{1,2} &= 1 \pm i \\ y &= x[C_1 \sin(\ln x) + C_2 \cos(\ln x)] \end{aligned}$$

Set 2.10:

12

$$\begin{aligned} m(m-1) - m &= 0 \\ m_{1,2} &= 0,\ 2 \\ u' &= \frac{\begin{vmatrix} 0 & x^2 \\ (3+x)xe^x & 2x \end{vmatrix}}{\begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix}} = -\frac{3+x}{2} x^2 e^x \\ u &= -\frac{x^3}{2} e^x \\ v' &= \frac{\begin{vmatrix} 1 & 0 \\ 0 & (3+x)xe^x \end{vmatrix}}{\begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix}} = \frac{3+x}{2} e^x \\ v &= (1 + \frac{x}{2}) e^x \\ y &= C_1 + (C_2 + e^x) x^2 \end{aligned}$$

13

$$\begin{aligned} 4\ddot{y} + 4\dot{y} - 3y &= 7e^{2t} - 15e^{3t} \\ \lambda_{1,2} &= -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{3}{4}} = \frac{1}{2}, -\frac{3}{2} \\ y_p &= Ae^{2t} + Be^{3t} \end{aligned}$$

Substituted into the equation yields:

$$\begin{aligned}21Ae^{2t}+45Be^{3t} &= 7e^{2t}-15e^{3t}\\ A &= \frac{1}{3}\quad B=-\frac{1}{3}\\ y(t) &= C_1e^{t/2}+C_2e^{-3t/2}+\frac{e^{2t}-e^{3t}}{3}\\ y(x) &= C_1\sqrt{x}+C_2x^{-3/2}+\frac{x^2-x^3}{3}\end{aligned}$$

16

$$\begin{aligned}m^2-1 &= 0\\ m_{1,2} &= \pm 1\\ u' &= \frac{\begin{vmatrix}0 & x\\ \frac{1}{x^4} & 1\end{vmatrix}}{\begin{vmatrix}\frac{1}{x} & x\\ -\frac{1}{x^2} & 1\end{vmatrix}}=-\frac{1}{2x^2}\\ u &= \frac{1}{2x}\\ v' &= \frac{\begin{vmatrix}\frac{1}{x} & 0\\ -\frac{1}{x^3} & \frac{1}{x^4}\end{vmatrix}}{\begin{vmatrix}\frac{1}{x} & x\\ -\frac{1}{x^2} & 1\end{vmatrix}}=\frac{1}{2x^4}\\ v &= -\frac{1}{6x^3}\\ y &= \left[C_1+\frac{1}{2x}\right]\frac{1}{x}+\left[C_2-\frac{1}{6x^3}\right]x=\\ & \frac{C_1}{x}+C_2x+\frac{1}{3x^2}\end{aligned}$$

Extra question:

$$\begin{aligned}\ddot{y}-5\dot{y}+6y &= te^{-4t}\\ \lambda_{1,2} &= \frac{5}{2}\pm\frac{1}{2}=2,3\\ y_p &= (At+B)e^{-4t}\end{aligned}$$

substituted in the equation yields:

$$42At-13A+42B=t$$
$$A=\frac{1}{42}\qquad B=\frac{13}{42^2}$$
$$\begin{aligned}t(t) &= C_1e^{2t}+C_2e^{3t}+(\frac{t}{42}+\frac{13}{1764})e^{-4t}\\ y(x) &= C_1x^2+C_2x^3+(\frac{\ln x}{42}+\frac{13}{1764})x^{-4}\end{aligned}$$