MATH 2F05 SUBSTITUTE 2^{nd} MIDTERM MARCH 16, 2005
Full credit given for three correct and complete answers.
For numerical answers, use *decimal* form with 4 significant digits.
Allowed: Formula summary, and a diskette. Duration: 50 minutes

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1. Consider the following motion of a point-like particle:

$$\mathbf{r} = [\ 2t\cos t,\ 2t\sin t,\ \sqrt{2t}\ ]$$

where t represents time. At what time will the particle reach the point $[0, \pi, \sqrt{\pi}]$? Find the speed, and tangential and normal acceleration of the particle at this point. What is the curvature and torsion of the corresponding path (at the same point).

2. Consider a two-dimensional region defined as all points which meet

$$x^2 + y^2 < 1 \quad \text{and} \quad y < \frac{1}{2}$$

(you must have a clear picture of it first). Find the total area of this region, its center of mass, and its moment of inertia with respect to the $y = -1$ line. Assume *uniform* mass density, with total mass equal to M. Hint: Do the integration in *regular* coordinates, don't switch to polar.

3. Consider a sphere of radius 1 centered at the origin (its equation is: $x^2 + y^2 + z^2 = 1$) and another one centered at [0,0,1], i.e. $x^2 + y^2 + (z-1)^2 = 1$. Find the volume of the three-dimensional region defined as all points which meet

$$x^2 + y^2 + z^2 < 1 \qquad \text{and} \qquad x^2 + y^2 + (z-1)^2 < 1$$

i.e. are *inside each* of the two spheres (their 'overlap', so to speak). Hint: Find the radius and location of the circle where the two spheres intersect. This will help you project the region into the x-y plane.

4. Using the technique of Frobenius, find the *first* basic solution to

$$(1-2x)xy'' + (1-6x)y' - 2y = 0$$

(extra marks given for the second basic solution, when found either by the same technique, or by 'reduction of order' a.k.a. V of P).

5. Evaluate
$$\int_C [yz - x^2,\ xz - y^2,\ xy - z^2] \bullet d\mathbf{r}$$
where C consists of the following five (joint) segments: $[3, -1, 4] \to [2, 0, -6] \to [-3, 2, 1] \to [7, -2, 4] \to [0, -3, 5] \to [2, 6, -3]$ (arrows implying straight-line connections).

6. Consider the following section of a conical shell of uniform mass density, with total mass M:
$$\sqrt{x^2 + y^2} = 2z \qquad \text{where} \qquad 1 < z < 3$$
Find its center of mass, and moment of inertia with respect to a straight line passing through $[0, 0, 2]$ and parallel to the x-axis.