BROCK UNIVERSITY

Final Examination: December 2007	Number of Pages: 4
Course: MATH 2P81	Number of students: 35
Date of Examination: Dec. 4, 2007	Number of Hours: 3
Time of Examination: 14:00 -17:00	Instructor: J. Vrbik

Three sheets of notes and the use of Maple are permitted.

No examination aids other than those specified on the examination scripts are permitted (this regulation does not preclude special arrangements being made for students with disabilities). Translation dictionaries (e.g. English-French) or other dictionaries (thesaurus, definitions, technical) are not allowed unless specified by the instructor and indicated on the examination paper.

Full credit given for 8 complete answers.

Numerical answers must be correct to 4 significant digits.

1. Consider playing a game with the following pay-off table:

$W =$	-3	0	7	17
$Pr:$	0.48	0.35	0.15	0.02

   (a) Find the expected value and standard deviation of W.

   (b) With the help of probability generating function (and Maple), find the probability of winning some money after 15 independent rounds of the game.

   (c) Using the normal approximation, find the probability of winning money after 15,000 independent rounds of this game.

2. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.47$, $\Pr(B) = 0.21$, $\Pr(C) = 0.83$ and $\Pr(D) = 0.55$, find

$$\Pr\left[(A \cup \bar{B} \cup \bar{C} \cup D) \cap (\bar{A} \cup B \cup C) \cap (\bar{A} \cup B \cup D)\right]$$

   Hint: First, find the probability of the complement.

3. Four girls and 16 boys are randomly divided into 4 teams of 5 players each. What is the probability that

   (a) the girls get separated (each playing for a different team),

   (b) they will all play for the same team,

   (c) each girl has at least one female team-mate?

4. Cards are dealt from a well shuffled regular deck, one by one, until we get the third spade. Let X be the total number of cards thus dealt. Find the *probability function* $f(i)$ of X (don't forget to specify its range), and (with the help of Maple) the corresponding mean and standard deviation. Hint: The last card must be a spade, how about the previous $i-1$ cards?

5. Suppose customers arrive at a rate of 23.7 per hour. Let X be the time of the third arrival, and Y is the time of the fifth arrival (from now). We know that X and $U \equiv Y - X$ are *independent* random variables having the $\gamma(3, \frac{1}{23.7})$ and $\gamma(2, \frac{1}{23.7})$ distribution, respectively. Find:

   (a) $\Pr(10 \text{ min } < Y < 20 \text{ min})$,

   (b) the correlation coefficient between X and Y, Hint: $Y = U + X$.

   (c) $\Pr(X < Y/2)$. Use the same hint.

6. Five dice are rolled (this constitutes *one* trial), repeatedly, until they show three or more *identical* numbers (a 'success'). Find

   (a) the expected number of trials, and the corresponding standard deviation,

   (b) the probability that we will need at least 9 trials to succeed (once),

   (c) the probability that we will need at least 9 trials to succeed for the *third time*.

7. A random variable X has the following probability generating function:

$$\left(\frac{2+z}{4-z}\right)^{12}$$

   Find

   (a) its expected value and standard deviation,

   (b) (with Maple's help) $\Pr(X > 10)$,

   (c) and the *moment* generating function of $2X - 3$.

8. X and Y have a bi-variate distribution described by the following joint probability density function:

$$f(x, y) = c \cdot e^{-y} \quad \text{for} \quad x > 0,\ y > x$$

zero otherwise. Find:

(a) the value of c,

(b) $\Pr(X + Y > 0.5)$,

(c) $\mathbb{E}(Y \mid X = 2)$,

(d) the (marginal) *distribution function* of Y.

9. Suppose we have 7 identically looking dice, 5 of them are regular, but two of them are biased in the following manner:

# of dots:	1	2	3	4	5	6
Pr:	$\frac{3}{48}$	$\frac{5}{48}$	$\frac{7}{48}$	$\frac{8}{48}$	$\frac{10}{48}$	$\frac{15}{48}$

We select one of them randomly, and roll it 8 times.

(a) What is the probability of getting at least 3 sixes?

(b) Given that the eight rolls resulted in 4, 6, 3, 6, 4, 5, 5, 6, what is the conditional probability of having selected one of the biased dice?

10. Two cards are dealt from a well-shuffled regular deck. X and Y is the number of aces and spades, respectively, found in this hand. Build a table of the bivariate distribution of X and Y. Also, compute:

(a) $\mathbb{E}[(X - Y)^2]$,

(b) $\mathbb{E}\left(\frac{1}{1+X} \mid Y = 1\right)$,

(c) $\text{Cov}(X, Y)$.

11. There are 10 blue, 13 red, 9 green and 7 yellow marbles in a box. Six marbles are randomly drawn, without replacement. Find:

    (a) the probability of getting at least 2 blue and (at the same time - this is *one* question) at least 2 red marbles,

    (b) the expected number of green marbles obtained in this sample, and the corresponding standard deviation,

    (c) the *correlation coefficient* between the number of red, and the number of yellow marbles in this sample.

12. Let X be a random variable with the following probability density function:

$$f(x) = \begin{cases} 0 & x \le 0 \\ c \cdot x^2 & 0 < x \le 2 \\ c \cdot e^{-x} & 2 < x \end{cases}$$

    Find:

    (a) the value of c,

    (b) the mean and standard deviation of this distribution.

    (c) Consider a random independent sample of size 392 from this distribution. Use the normal approximation to find $\Pr(1.5 < \bar{X} < 1.6)$.