1. $--$

   (a)

$$\begin{aligned}\mu &= -3\times 0.48+7\times 0.15+17\times 0.02=-0.05\\ \sigma &= \sqrt{(-3)^2\times 0.48+7^2\times 0.15+17^2\times 0.02-(-0.05)^2}=4.177\end{aligned}$$

   (b) Expand

$$(0.48z^{-3}+0.35+0.15z^{7}+0.02z^{17})^{15}$$

   and add coefficients from z to z^{255}, getting 42.22%.

   (c)

$$\frac{1}{4.177\sqrt{2\pi\times 15000}}\int_{0.5}^{\infty}\exp\left(-\frac{(x-15000\times(-0.05))^2}{2\times 4.177^2\times 15000}\right)dx=7.118\%$$

2. First the complement:

$$\Pr\left[(\bar{A}\cap B\cap C\cap\bar{D})\cup(A\cap\bar{B}\cap\bar{C})\cup(A\cap\bar{B}\cap\bar{D})\right]=$$

$$\Pr(\bar{A}\cap B\cap C\cap\bar{D})+\Pr(A\cap\bar{B}\cap\bar{C})+\Pr(A\cap\bar{B}\cap\bar{D})-\Pr(A\cap\bar{B}\cap\bar{C}\cap\bar{D})=$$

$$0.53\times 0.21\times 0.83\times 0.45+0.47\times 0.79\times 0.17+0.47\times 0.79\times 0.45-$$

$$-0.47\times 0.79\times 0.17\times 0.45=0.24337$$

   implying that the answer is $1-0.24337=75.66\%$

3. $--$

   (a)

$$\frac{\binom{16}{4,4,4,4}}{\binom{20}{5,5,5,5}/4!}=12.90\%$$

   (b)

$$\frac{\binom{16}{1,5,5,5}/3!}{\binom{20}{5,5,5,5}/4!}=0.4128\%$$

   (c)

$$\frac{\binom{4}{2,2}/2\times\binom{16}{3,3,5,5}/2}{\binom{20}{5,5,5,5}/4!}+b)=12.80\%$$

4.

$$f(i)=\frac{\binom{13}{2}\times\binom{39}{i-3}}{\binom{52}{i-1}}\times\frac{11}{53-i}$$

where $3 \le i \le 42$.

$$\begin{aligned}\mu &= \sum_{i=3}^{42} i \times f(i) = 11.357 \\ \sigma &= \sqrt{\sum_{i=3}^{42} i^2 \times f(i) - 11.357^2} = 4.817\end{aligned}$$

5. – –

(a)

$$\frac{23.7^5}{4!}\int_{1/6}^{1/3} x^4 \exp(-x \times 23.7)dx = 53.31\%$$

(b)

$$\begin{aligned}\mathrm{Cov}(X, U + X) &= 0 + \frac{3}{23.7} \\ \mathrm{Var}(X) &= \frac{3}{23.7} \\ \mathrm{Var}(Y) &= \frac{5}{23.7} \\ \rho_{xy} &= \frac{\frac{3}{23.7}}{\sqrt{\frac{3}{23.7} \times \frac{5}{23.7}}} = \sqrt{\frac{3}{5}} = 0.7746\end{aligned}$$

(c)

$$\begin{aligned}\Pr(X &< Y/2) = \Pr(X < \frac{X+U}{2}) = \Pr(X < U) = \\ &\frac{23.7^5}{2!}\int_0^\infty \int_x^\infty x^2 \exp(-x \times 23.7) \times u \exp(-u \times 23.7) du dx = 31.25\%\end{aligned}$$

6. Probability of a success is

$$p = \frac{6 + 6 \times 5 \times 5 + 6 \times 5 \times \binom{5}{2} + 6 \times \binom{5}{2} \times \binom{5}{3,1,1}}{6^5} = \frac{23}{108}$$

(quintuplet + quadruplet + triplet and pair + triplet and two singlets).

(a)

$$\begin{aligned}\mu &= \frac{1}{p} = 4.696 \\ \sigma &= \sqrt{\frac{1}{p}(\frac{1}{p} - 1)} = 4.166\end{aligned}$$

(b)

$$(1-p)^8 = 14.72\%$$

(c)

$$\sum_{i=0}^{2} \binom{8}{i} p^i (1-p)^{8-i} = 76.77\%$$

7. ––

(a) Since

$$M(t) = \left(\frac{2+e^t}{4-e^t}\right)^{12}$$

we get

$$\begin{aligned} \mu &= \left.\frac{dM(t)}{dt}\right|_{t=0} = 8 \\ \sigma &= \sqrt{\left.\frac{d^2M(t)}{dt^2}\right|_{t=0} - 8^2} = 2.828 \end{aligned}$$

(b) Taylor-expanding

$$\begin{aligned} \left(\frac{2+z}{4-z}\right)^{12} &= 0.0002 + 0.0022z + 0.0096z^2 + 0.0273z^3 + 0.0570z^4 + 0.0932z^5 \\ &\quad +0.1251z^6 + 0.1423z^7 + 0.1408z^8 + 0.1233z^9 + 0.09732z^{10} + ... \end{aligned}$$

in z and adding coefficients from z^0 to z^{10}, we get 0.8185. The answer: $1 - 0.8185 = 18.15\%$.

(c)

$$e^{-3t} \times M(2t) = e^{-3t} \left(\frac{2+e^{2t}}{4-e^{2t}}\right)^{12}$$

8. ––

(a)

$$c \int_0^\infty \int_x^\infty e^{-y} dy dx = c$$

This implies that $c = 1$.

(b)

$$1 - \int_0^{1/4} \int_x^{1/2-x} e^{-y} dy dx = 95.11\%$$

(c)

$$\frac{\int_2^\infty y e^{-y} dy}{\int_2^\infty e^{-y} dy} = 3$$

(d)

$$f(y) = \int_0^y e^{-y} dx = ye^{-y}$$

for $y > 0$. This implies that

$$F(y) = \int_0^y ue^{-u} du = -e^{-u}(1+u)\Big|_{u=0}^{y} = 1 - e^{-y}(1+y)$$

for $y > 0$.

9. __

(a)

$$\frac{5}{7}\sum_{i=3}^{8} \tbinom{8}{i}(\tfrac{1}{6})^i(\tfrac{5}{6})^{8-i} + \frac{2}{7}\sum_{i=3}^{8} \tbinom{8}{i}(\tfrac{15}{48})^i(\tfrac{33}{48})^{8-i} = 23.34\%$$

(b)

$$\frac{\frac{2}{7}(\frac{15}{48})^3(\frac{10}{48})^2(\frac{8}{48})^2\frac{7}{48}}{\frac{5}{7}\times(\frac{1}{6})^8 + \frac{2}{7}(\frac{15}{48})^3(\frac{10}{48})^2(\frac{8}{48})^2\frac{7}{48}} = 78.28\%$$

10.

$Y \downarrow \ X \rightarrow$	0	1	2
0	$\frac{\binom{36}{2}}{\binom{52}{2}} = \frac{105}{221}$	$\frac{36\times 3}{\binom{52}{2}} = \frac{18}{221}$	$\frac{\binom{3}{2}}{\binom{52}{2}} = \frac{1}{442}$
1	$\frac{36\times 12}{\binom{52}{2}} = \frac{72}{221}$	$\frac{12\times 3 + 36}{\binom{52}{2}} = \frac{12}{221}$	$\frac{3}{\binom{52}{2}} = \frac{1}{442}$
2	$\frac{\binom{12}{2}}{\binom{52}{2}} = \frac{11}{221}$	$\frac{12}{\binom{52}{2}} = \frac{2}{221}$	0

(a)

$$1^2\times\left(\frac{18}{221} + \frac{1}{442} + \frac{72}{221} + \frac{2}{221}\right) + 2^2\times\left(\frac{1}{442} + \frac{11}{221}\right) = \frac{277}{442} = 0.6267$$

(b)

$$\frac{\frac{1}{21}\times\frac{72}{221} + \frac{1}{2}\times\frac{12}{221} + \frac{1}{3}\times\frac{1}{442}}{\frac{72}{221} + \frac{12}{221} + \frac{1}{442}} = \frac{31}{273} = 0.11355$$

(c)

$$-2\times\left(\tfrac{13}{52}\times\tfrac{4}{52} - \tfrac{1}{52}\right)\tfrac{52-2}{52-1} = 0$$

11. __

(a)

$$\frac{\binom{10}{2}\binom{13}{2}\binom{16}{2}+\binom{10}{3}\binom{13}{2}16+\binom{10}{4}\binom{13}{2}}{\binom{39}{6}}+$$
$$+\frac{\binom{10}{3}\binom{13}{3}+\binom{10}{2}\binom{13}{3}16+\binom{10}{2}\binom{13}{4}}{\binom{39}{6}}=26.35\%$$

(b)

$$\begin{aligned} 6\times\frac{9}{39} &= \frac{18}{13}=1.3846 \\ \sqrt{6\times\frac{9}{39}\times\frac{30}{39}\times\frac{39-6}{39-1}} &= 0.9617 \end{aligned}$$

(c)

$$\frac{-6\times\frac{13}{39}\times\frac{7}{39}\times\frac{39-6}{39-1}}{\sqrt{6\times\frac{13}{39}\times\frac{26}{39}\times\frac{39-6}{39-1}\cdot 6\times\frac{7}{39}\times\frac{32}{39}\times\frac{39-6}{39-1}}}=\frac{-\sqrt{7}}{8}=-0.33072$$

12. __

(a)

$$c=\frac{1}{\int_0^2 x^2dx+\int_2^\infty \exp(-x)dx}=0.35689$$

(b)

$$\begin{aligned} \mu &= c\int_0^2 x^3dx+c\int_2^\infty x\exp(-x)dx=1.5725 \\ \sigma &= \sqrt{c\int_0^2 x^4dx+c\int_2^\infty x^2\exp(-x)dx-\mu^2}=0.54253 \end{aligned}$$

(c)

$$\frac{\sqrt{392}}{\sqrt{2\pi}\sigma}\int_{1.5}^{1.6}\exp\left(-\frac{(x-\mu)^2}{2\times\sigma^2/392}\right)dx=83.81\%$$