BROCK UNIVERSITY

Final Examination: December 2008	Number of Pages: 4
Course: MATH 2P81	Number of students: 32
Date of Examination: Dec. 12, 2008	Number of Hours: 3
Time of Examination: 19:00 - 22:00	Instructor: J. Vrbik

Two sheet of notes, and the use of Maple, are permitted.

No examination aids other than those specified on the examination scripts are permitted (this regulation does not preclude special arrangements being made for students with disabilities). Translation dictionaries (e.g. English-French) or other dictionaries (thesaurus, definitions, technical) are not allowed unless specified by the instructor and indicated on the examination paper.

Full credit given for 8 complete answers.

Numerical answers must be correct to 4 significant digits.

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1. Given that

$$\begin{aligned} P(A) &= 0.62, \quad P(B) = 0.61, \quad P(C) = 0.67 \\ P(A \cap B) &= 0.33, \quad P(A \cap C) = 0.32, \quad P(B \cap C) = 0.36 \\ &\text{and } P(A \cap B \cap C) = 0.15 \end{aligned}$$

find

$$\Pr\left[(A \cap \bar{B}) \cup (\bar{A} \cap C) \cup \overline{B \cup C}\right]$$

2. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.42$, $\Pr(B) = 0.23$, $\Pr(C) = 0.81$ and $\Pr(D) = 0.54$, find

$$\Pr\left[(A \cup \bar{B} \cup \bar{C} \cup D) \cap (\bar{A} \cup B \cup C) \cap (\bar{A} \cup B \cup D)\right]$$

3. Four girls and 16 boys are randomly divided into 5 teams of 4 players each. What is the probability that

   (a) the girls get separated (each playing for a different team),

   (b) they will all play for the same team,

   (c) each girl has at least one female team-mate?

4. Let X and Y be two integer-valued random variables with the following joint probability function:

   $f_{xy}(i,j) = c \cdot (2-j)$ where $-2 \le i \le 2$ while $1 - |i| \le j \le 1$

   Find:

   (a) the value of c,

   (b) $\mathbb{E}(Y)$ and $\mathrm{Var}(Y)$,

   (c) $\mathrm{Cov}(X, Y)$,

   (d) $\mathbb{E}(\frac{1}{2+Y} \mid X = 2)$.

5. Let X be a random variable with the following probability density function:

$$f(x) = \begin{cases} c \cdot x^2 & 0 < x \le 2 \\ c \cdot e^{-x} & 2 < x \\ 0 & \text{otherwise} \end{cases}$$

   Find:

   (a) the value of c,

   (b) the distribution function $F(x)$,

   (c) $\Pr(1 < X < 2.5)$,

   (d) the mean, median and variance of this distribution.

6. Consider paying \$5 to play the following game: a die is rolled 10 times, and you get paid \$1 for the first six (if it ever occurs), \$2 for the second six (if it occurs), \$4 for the third one, etc. (the amount always doubles - for the 10^{th} six, should you be that lucky, you would get \$1024, on top of the \$1023 you have already received). Find the expected net win, and the corresponding standard deviation, in one round of this game. What is the probability of breaking even after *three* rounds?

7. There are 21 red, 13 blue and 4 yellow marbles in a box. You pay \$10 to draw randomly 3 marbles from this box (without replacement) and receive \$1 for each red, \$5 for each blue and \$10 for each yellow marble. Compute the expected value and standard deviation of your net win. Also, what is the probability of winning (net) at least \$10?

8. There are 21 red, 13 blue and 4 yellow marbles in a box. A marble is randomly drawn, its color noted, and then the marble is returned to the box, together with 3 *extra* marbles of the same color as the marble drawn. This is repeated two *more* times (for the total of three draws). Find the distribution of the number of red marbles drawn, its expected value and standard deviation. Given that the last marble drawn was red, what is the conditional probability that the first marble was either blue or yellow?

9. Let X and Y be two *independent* random variables, both having the geometric distribution with $p = \frac{1}{6}$.

   (a) Compute $\Pr(X > Y)$. Hint: Compute $\Pr(X = Y)$ first.

   (b) What is the expected value and standard deviation of $Y - 3X + 2$?

   (c) Find $\text{Cov}(Y - 3X + 2, 5X - 2Y + 1)$.

   (d) Compute $\Pr(X + Y > 10)$.

10. Customer arrive at a store randomly, at an average rate of 13.2 per hour. The store opens at 9:00.

    Compute the probability that

    (a) at least 5 customers arrive between 9:00 and 9:25,

    (b) the third customer of the day arrives between 9:10 and 9:20.

    (c) Find the expected time of arrival of the third customer, and the corresponding standard deviation.

11. Two independent random variables X and Y have the following moment generating function

$$
\begin{aligned}
M_x(t) &= \frac{e^{4t}}{(1-3t)^2} \\
M_y(t) &= \frac{e^{4t}}{(3-2e^t)^4}
\end{aligned}
$$

respectively. Find

(a) the mean and variance (individually) of each X and Y,

(b) the moment generating function of $3 - \frac{X}{2} + 2Y$,

(c) $\Pr(Y > 21)$.

12. Consider the following game: 6 dice are rolled and 10 coins flipped; your have to pay \$3 for each one (the face with one dot), but collect \$3 for every six. Similarly, you pay \$1 for each Head and receive \$1 for each Tail. What is the expected net win of this game, and the corresponding standard deviation. Find the moment generating function of the net win. Hint: first compute the MGF of the net win in a single roll, and the MGF of the net win in a single flip.