BROCK UNIVERSITY

Substitute Final Examination: December 2008	Number of Pages: 4
Course: MATH 2P81	Number of students: 43
Date of Examination: Dec. 21, 2009	Number of Hours: 3
Time of Examination: 8:00-12:00	Instructor: J. Vrbik

Two sheet of notes, and the use of Maple, are permitted.

No examination aids other than those specified on the examination scripts are permitted (this regulation does not preclude special arrangements being made for students with disabilities). Translation dictionaries (e.g. English-French) or other dictionaries (thesaurus, definitions, technical) are not allowed unless specified by the instructor and indicated on the examination paper.

Full credit given for 7 complete answers.

Numerical answers must be correct to 4 significant digits.

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1. Given that

$$\begin{aligned} P(A) &= 0.50, \quad P(B) = 0.54, \quad P(C) = 0.53, \quad P(A \cap B) = 0.21, \\ P(A \cap C) &= 0.25, \quad P(B \cap C) = 0.30 \text{ and } P(A \cap B \cap C) = 0.12 \end{aligned}$$

   find

$$\Pr\left[\overline{A \cap B} \cap \overline{A \cap C} \cap \overline{B \cap C}\right]$$

2. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.42$, $\Pr(B) = 0.23$, $\Pr(C) = 0.81$ and $\Pr(D) = 0.54$, find

$$\Pr\left[(A \cap \overline{B \cap D}) \cup (\overline{A \cap C} \cap D)\right]$$

   Hint: In both questions, do the complement first.

3. Four males and eight females are randomly seated at a round table. What is the probability that

   (a) each female has one male and one female neighbour,

   (b) three males sit together (in three consecutive chairs), while the fourth one is separated from them by at least one female,

   (c) at least 5 females sit together (in five or more consecutive chairs).

4. Four dice are rolled, followed by dealing as many cards as the largest number of identical outcomes obtained (deal 1 card when the dice show four distinct numbers or 'singlets', 2 cards when there is at least one pair, 3 cards for a triplet, and 4 cards for a quadruplet). Compute

   (a) the probability of having to deal more than 2 cards,

   (b) the probability of finding (exactly) one ace and (exactly) one heart, among the cards dealt,

   (c) the conditional probability of having obtained a triplet (in the first part of the experiment), given that the resulting hand contains (exactly) one ace and (exactly) one heart.

5. Rolling 7 dice constitutes a single trial, and getting at least 3 sixes is considered a success. Let X be the number of trials required to get five successes (and stop rolling). Compute:

   (a) the mean and standard deviation of $3 - 2X$,

   (b) the third factorial moment of X,

   (c) $\Pr(40 \le X \le 60)$.

6. The joint probability function of X and Y is given by

$$f_{xy}(i, j) = c \cdot i \cdot (4 - j)$$

   where c is an appropriate constant, and i and j are both *positive* (i.e. *greater* than 0) integers, such that $2i + 3j < 12$. Are X and Y independent (substantiate your answer)? Also, find

   (a) the value of c,

   (b) the correlation coefficient between X and Y,

   (c) $\mathbb{E}\left(\frac{1}{X}|Y = 2\right)$.

7. Customers arrive at a rate of 11.72 per hour. The store opens at 8:00. Find:

   (a) The expected time of arrival of the 15^{th} customer (present the answer in the hour:minute:second format) and the corresponding standard deviation (in minutes and seconds). Also, the *median* time of his/her arrival (in the same format).

   (b) Probability that the 15^{th} customer arrives between 8:57 and 9:12.

   (c) Probability of getting more than 15 customers between 8:57 and 9:12.

8. Let X be a random variable with the following *distribution function*:

$$F(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{x^2}{2} & 0 < x \le 1 \\ 1 - \dfrac{(2-x)^2}{2} & 1 < x \le 2 \\ 1 & x > 2 \end{cases}$$

   (a) Find the expected value, standard deviation, skewness and kurtosis of X.

   (b) Suppose we draw a random independent sample of size 50 from this distribution. Using the Normal approximation, find $\Pr(0.87 < \bar{X} < 1.06)$, where $\bar{X}$ is the corresponding sample mean.

9. There are 9 red, 12 blue and 8 green marbles in a box. Consider randomly drawing marbles from this box, one by one, without replacement. Let X be the number of draws needed to get 3 red marbles (including the last draw - the one which yielded the 3^{rd} red marble).

   (a) Find the *probability function* of X, and its range. Hint: The last, say i^{th} draw, must result in a red marble. How about the previous $i-1$ draws?

   (b) Compute the mean and standard deviation of X,

   (c) and $\Pr(X > 15)$.

10. Consider paying \$20 to play the following game: There are 9 red, 12 blue and 8 green marbles in a box. Seven of them are randomly drawn, one by one, *with* replacement (after each draw, the marble is returned to the box and mixed with the other marbles, before selecting the next one). The player receives \$5 for each red marble and \$3 for each blue marble drawn. What is the probability of winning (net) at least \$10 in

    (a) 1 round of this game (compute *directly*, *not* using the probability generating function),

    (b) 10 independent rounds of this game (use the probability generating function),

    (c) 100 independent rounds of this game (use the Normal approximation).

11. Let X_1, X_2, ... X_{12} be a random independent sample (yet to be taken) from an exponential distribution with the mean of 1 hour and 23 minutes. Compute

    (a) the probability that their *sample mean* will be less than 1 hour (hint: express in terms of the *sum* of these - what is the corresponding distribution?),

    (b) the smallest one of these 12 random variables will be less than 15 minutes,

    (c) the covariance between $3X_1+2X_2-4X_3+1$ and $X_1-3X_2+5X_3-2$ (express in hour^2). Hint: use the distributive law of covariance.