BROCK UNIVERSITY

Final Examination: December 2011	Number of Pages: 4
Course: MATH 2P81	Number of students: 42
Date of Examination: Dec. 19, 20011	Number of Hours: 3
Time of Examination: 9:00-12:00	Instructors: J. Vrbik

Two sheet of notes and the use of Maple are permitted.

No examination aids other than those specified on the examination scripts are permitted (this regulation does not preclude special arrangements being made for students with disabilities). Translation dictionaries (e.g. English-French) or other dictionaries (thesaurus, definitions, technical) are not allowed unless specified by the instructor and indicated on the examination paper.

Full credit given for 7 complete answers.

Numerical answers must be correct to 4 significant digits.

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1. Given that

$$\begin{aligned} P(A) &= 0.50, \quad P(B) = 0.54, \quad P(C) = 0.53, \quad P(A \cap B) = 0.21, \\ P(A \cap C) &= 0.25, \quad P(B \cap C) = 0.30 \text{ and } P(A \cap B \cap C) = 0.12 \end{aligned}$$

find

(a)
$$\Pr\left[\bar{A} \cup B \cup C\right]$$

(b)
$$\Pr\left[\bar{A} \cap (B \cup C)\right]$$

2. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.42$, $\Pr(B) = 0.23$, $\Pr(C) = 0.81$ and $\Pr(D) = 0.54$, find

(a)
$$\Pr\left[A \cup \bar{B} \cup C \cup \bar{D}\right]$$

(b)
$$\Pr\left[A \cap (B \cup C) \cap (A \cup C \cup D)\right]$$

3. Four males (we will call them Mr. A, B, C and D) and nine females are randomly seated in a row. What is the probability that

   (a) Mr. A and B sit next to each other, while (at the same time - this is one question) Mr C and D sit apart (separated by at least one person),

   (b) all males have only female neighbours.

4. Consider paying \$11 to play the following game: four dice are rolled and you get paid \$5 for *each* pair, \$50 for a triplet and \$500 for a quadruplet (nothing if all four numbers are distinct). Find

   (a) the expected value and standard deviation of your *net* win (in a single round),

   (b) the probability of losing money after four rounds of this game (use the corresponding PGF; you will need Maple to complete the computation).

5. Consider playing a game with the following net pay-off

$W =$	-3	1	2	5	10
Pr:	0.42	0.32	0.16	0.08	0.02

   Find

   (a) the mean, standard deviation and kurtosis of this distribution,

   (b) the probability of winning money after 10,000 rounds of this game (use the Normal approximation).

6. X and Y are *independent* random variables having the $\mathcal{P}(\Lambda = 3)$ and $\mathcal{NB}\left(k = 5, p = \frac{1}{3}\right)$ distribution (Poisson and Negative Binomial), respectively. Find

   (a) the PGF of $2X + 3Y - 5$,

   (b) $\Pr(2X + 3Y > 30)$.

7. Customers arrive, randomly, at the average rate of 5.3 per hour. It is 8:17 now. What is the probability that

   (a) there will be more than 10 customers arriving between now and 9:52,

   (b) the 5^{th} customer from now will arrive between 9:00 and 9:15.

8. Consider three 'black' boxes, the first contains 12 marbles (4 of them red), the second one has 14 marbles (8 of them red) and the last one has has 17 marbles (14 of them red). One of these boxes is selected at random and 4 marbles are drawn from it (without replacement). Find the probability that

   (a) at least 3 of the drawn marbles will be red,

   (b) it was the last box which was selected, given that *exactly* 3 of the drawn marbles were red.

9. Consider the usual tri-nomial expansion of

$$\left(\frac{1}{18} - 3x^2 + \frac{x \cdot y}{5}\right)^{1000}$$

   (a) How many terms are there in this expansion?

   (b) What is the coefficient of $x^{1137}y^{519}$?

10. Three cards are randomly dealt from a standard deck of 52 cards. Let U be the number of spades, V the number of diamonds and W the number of aces obtained. Find:

   (a) the mean and standard deviation of $2U - 5V + 4W - 3$,

   (b) the conditional distribution of U given that $W = 1$.

11. Consider playing the following game: 8 cards are randomly dealt from a standard deck, and you receive \$10 for each ace, \$5 for each face card (J, Q, K), but have to pay \$3 for every other card. Find

   (a) the expected value of your net win (may be negative) and the corresponding standard deviation,

   (b) the probability of winning some money (in a single round). Hint: the complement may be easier.