

1.

(a)

$$\begin{aligned} \Pr[\bar{A} \cup B \cup C] &= 1 - \Pr(A \cap \bar{B} \cap \bar{C}) = \\ &= 1 - \Pr(A \cap \bar{B}) + \Pr(A \cap \bar{B} \cap C) = \\ &= 1 - \Pr(A) + \Pr(A \cap B) + \Pr(A \cap C) - \Pr(A \cap B \cap C) = \\ &= 1 - .5 + .21 + .25 - .12 = 0.84 \end{aligned}$$

(b)

$$\begin{aligned} \Pr[\bar{A} \cap (B \cup C)] &= \Pr(B \cup C) - \Pr[(B \cup C) \cap A] = \\ &= \Pr(B) + \Pr(C) - \Pr(B \cap C) - \Pr[(A \cap B) \cup (A \cap C)] = \\ &= \Pr(B) + \Pr(C) - \Pr(B \cap C) - \Pr(A \cap B) - \Pr(A \cap C) + \Pr(A \cap B \cap C) = \\ &= .54 + .53 - .30 - .21 - .25 + .12 = 0.43 \end{aligned}$$

2.

(a)

$$\begin{aligned} \Pr[A \cup \bar{B} \cup C \cup \bar{D}] &= 1 - \Pr(\bar{A}) \Pr(B) \Pr(\bar{C}) \Pr(D) \\ &= 1 - 0.58 \times 0.23 \times 0.19 \times 0.54 = 0.98631 \end{aligned}$$

(b)

$$\begin{aligned} \Pr[A \cap (B \cup C) \cap (A \cup C \cup D)] &= \Pr[A \cap (B \cup C)] = \\ &= \Pr(A) \times [\Pr(B) + \Pr(C) - \Pr(B) \Pr(C)] \\ &= 0.42 \times (0.23 + 0.81 - 0.23 \times 0.81) = 0.35855 \end{aligned}$$

3.

(a)

$$\frac{2 \times 12! - 2^2 \times 11!}{13!} = 12.82\%$$

(b)

$$\frac{4 \times 9 \times 8 \times 7 \times 9! + 9 \times 8 \times 7 \times 6 \times 9!}{13!} = 29.37\%$$

4.

(a)

outcome	nothing	1 pair	2 pairs	triplet	quadruplet
net win	-11	-6	-1	39	489
Pr	$\frac{6 \cdot 5 \cdot 4 \cdot 3}{6^4} = \frac{5}{18}$	$\frac{6 \cdot \binom{4}{2} \cdot 5 \cdot 4}{6^4} = \frac{5}{9}$	$\frac{\binom{6}{2} \cdot \binom{4}{2}}{6^4} = \frac{5}{72}$	$\frac{6 \cdot \binom{4}{3} \cdot 5}{6^4} = \frac{5}{54}$	$\frac{6}{6^4} = \frac{1}{216}$

$$\mu = -11 \times \frac{5}{18} - 6 \times \frac{5}{9} - 1 \times \frac{5}{72} + 39 \times \frac{5}{54} + 489 \times \frac{1}{216} = -\frac{7}{12}$$

$$\sqrt{\left(-11 + \frac{7}{12}\right)^2 \cdot \frac{5}{18} + \left(-6 + \frac{7}{12}\right)^2 \cdot \frac{5}{9} + \left(-1 + \frac{7}{12}\right)^2 \cdot \frac{5}{72} + \left(39 + \frac{7}{12}\right)^2 \cdot \frac{5}{54} + \left(489 + \frac{7}{12}\right)^2 \cdot \frac{1}{216}}$$

$$= \$36.07$$

(b)

$$\left(z^{-11} \times \frac{5}{18} + z^{-6} \times \frac{5}{9} + z^{-1} \times \frac{5}{72} + z^{39} \times \frac{5}{54} + z^{489} \times \frac{1}{216}\right)^4 \rightarrow 66.42\%$$

5.

(a)

$$\frac{-0.02}{\sqrt{8.7396}}$$

$$3.778$$

(b)

$$\frac{1}{\sqrt{2\pi \times 8.7396 \times 10000}} \int_{0.5}^{\infty} \exp\left(-\frac{(x + 0.02 \times 10000)^2}{2 \times 8.7396 \times 10000}\right) dx = 24.88\%$$

6.

(a)

$$\exp\left(3(z^2 - 1)\right) \left(\frac{z^3}{3 - 2z^3}\right)^5 z^{-5}$$

(b)

$$76.46\%$$

7.

(a)

$$\Lambda = (60 + 52 - 17) \times 5.3/60 = 8.3917$$

$$1 - e^{-8.3917} \sum_{i=0}^{10} \frac{8.3917^i}{i!} = 22.48\%$$

(b)

$$\Lambda_1 = (60 - 17) \times 5.3/60 = 3.7983$$

$$\Lambda_2 = (75 - 17) \times 5.3/60 = 5.1233$$

$$e^{-3.7983} \sum_{i=0}^4 \frac{3.7983^i}{i!} - e^{-5.1233} \sum_{i=0}^4 \frac{5.1233^i}{i!} = 24.90\%$$

8.

(a)

$$\frac{\sum_{i=3}^4 \frac{\binom{4}{i} \binom{8}{4-i}}{\binom{12}{4}} + \sum_{i=3}^4 \frac{\binom{8}{i} \binom{6}{4-i}}{\binom{14}{4}} + \sum_{i=3}^4 \frac{\binom{14}{i} \binom{3}{4-i}}{\binom{17}{4}}}{3}$$

$$= \frac{\frac{1}{15} + \frac{58}{143} + \frac{299}{340}}{3} = 45.06$$

(b)

$$\frac{\frac{\binom{14}{3} \binom{3}{1}}{\binom{17}{4}}}{\frac{\binom{4}{3} \binom{8}{1}}{\binom{12}{4}} + \frac{\binom{8}{3} \binom{6}{1}}{\binom{14}{4}} + \frac{\binom{14}{3} \binom{3}{1}}{\binom{17}{4}}} = 53.405\%$$

9.

(a)

$$\binom{1002}{2} = 501,501$$

(b)

$$\binom{1000}{519} \binom{481}{309} \left(\frac{1}{18}\right)^{172} (-3x^2)^{309} \left(\frac{x \cdot y}{5}\right)^{519} = -456.43x^{1137}y^{519}$$

since  $\frac{1137 - 519}{2} = 309$

10.

(a)

$$2U - 5V + 4W - 3$$

$$3\left(2 \times \frac{1}{4} - 5 \times \frac{1}{4} + 4 \times \frac{1}{13}\right) - 3 = -4.3269$$

$$\sqrt{3\left(4 \times \frac{1}{4} \times \frac{3}{4} + 25 \times \frac{1}{4} \times \frac{3}{4} + 16 \times \frac{1}{13} \times \frac{12}{13} + 20 \times \frac{1}{4} \times \frac{1}{13}\right) \frac{49}{51}} = 4.4784$$

(b)

$U \mid W = 1$	0	1	2	3
Pr	$\frac{3 \binom{36}{2}}{4 \binom{48}{2}} =$	$\frac{3 \cdot 12 \binom{36}{1} + \binom{36}{2}}{4 \binom{48}{2}} =$	$\frac{3 \binom{12}{2} + 12 \cdot 36}{4 \binom{48}{2}} =$	$\frac{\binom{12}{2}}{4 \binom{48}{2}} =$
	$\frac{315}{752}$	$\frac{321}{752}$	$\frac{105}{752}$	$\frac{11}{752}$

11.

(a)

$$10X + 5Y - 3(8 - X - Y) = 13X + 8Y - 24$$

$$8\left(13 \cdot \frac{1}{13} + 8 \cdot \frac{3}{13}\right) - 24 = -1.2308$$

$$\sqrt{8\left(13^2 \cdot \frac{1}{13} \cdot \frac{12}{13} + 8^2 \cdot \frac{3}{13} \cdot \frac{10}{13} - 26 \cdot 8 \cdot \frac{1}{13} \cdot \frac{3}{13}\right) \frac{44}{51}} = 11.651$$

(b)

$$1 - \frac{\binom{4}{1}\binom{12}{1}\binom{36}{6} + \binom{4}{1}\binom{36}{7} + \binom{12}{3}\binom{36}{5} + \binom{12}{2}\binom{36}{6} + \binom{12}{1}\binom{36}{7} + \binom{36}{8}}{\binom{52}{8}} = 37.70\%$$