1. Consider a random independent sample of size 712 from a distribution with the following pdf

$$f(x) = \frac{c}{1+x} \qquad 0 < x < 1$$

where c is an appropriate constant (you must first establish its value). Using the Normal approximation, find $\Pr(\overline{X} < 0.45)$.

2. Three girls and five boys are randomly seated at a round table. Find the probability that each girl has only male neighbors. (Hint: It may be easier to deal with the complement).

3. Given that $f_{XY}(i, j) = c(i + j)$ is a probability function of a bivariate discrete distribution, where i and j are two non-negative (i.e. including zero) integers such that $i^2 + j^2 \leq 9$, find

   (a) the value of c

   (b) marginal distribution of X

   (c) $\mathbb{E}(Y \mid X = 1)$

   (d) $\Pr(X + Y \leq 2)$.

4. If X and Y have the bivariate Normal distribution with $\mu_X = -1.3$, $\mu_Y = 12$, $\sigma_X = 2.4$, $\sigma_Y = 31$, and $\rho = 0.93$, calculate

   (a) $\Pr(Y > 15|X = -1.1)$

   (b) $\Pr(Y + 10X > 0)$.

5. Three cards are randomly dealt from an ordinary deck (of 52 cards) and, according to the number of spades obtained, a regular die is rolled that many times (possibly not at all). Compute

   (a) the probability of getting exactly 1 six

   (b) the conditional probability of having dealt at least 2 spades given that (exactly) 1 six was observed.

6. Consider the following pdf of a (*continuous*) bivariate distribution

$$f(x,y) = \frac{c\,x\,y}{x^2+y^2} \qquad \begin{cases} x>0 \\ y>0 \\ x^2+y^2<9 \end{cases}$$

Find

(a) the value of c

(b) marginal pdf of X

(c) conditional pdf of Y given that $X = 2$

(d) $\mathbb{E}(X^2Y)$.

Hint: When integrating, you may like to use polar coordinates (note that the region of definition is a quarter-circle).

In the next **two** questions, consider a continuous distribution with the following probability density function:

$$f(x) = \frac{1}{2\sqrt{x}} \qquad 0 < x < 1$$

7. Find its mean and standard deviation.

   Also, for a random independent sample of size 218 from this distribution, *approximate*

   $$\Pr\{\overline{X} < 0.29\}.$$

8. Find its median and quartile deviation (half the distance between the two quartiles).

9. Let A, B, C and D be four *independent* events having the probability of 0.21, 0.36, 0.18 and 0.62 respectively. Compute $\Pr\{(A \cup \overline{B}) \cap (C \cup \overline{D}) \cap (A \cup \overline{C})\}$.

   Hint: It may be easier to deal with the compliment.

10. Consider rolling 12 dice. What is the probability of getting (exactly) two *quadruplets* of identical numbers?

11. Let a bivariate probability density function be given by the following formula:

$$f(x,y) = cx^2 \qquad \text{when} \begin{cases} x > 0 \\ y > 0 \\ x + y < 1 \end{cases}$$

(zero otherwise). Find:

(a) The value of c,

(b) $\Pr\{X > Y\}$,

(c) the marginal probability density function of Y,

(d) $\mathbb{E}\{\frac{1}{X} \mid Y = \frac{1}{2}\}$.

12. Let X and Y have the bivariate normal distribution with $\mu_X = 3$, $\mu_Y = -2$, $\sigma_X = \sigma_Y = 5$ and $\rho = 0$. Compute:

(a) $\Pr\{(-10 < X < 10) \cap (-10 < Y < 10)\}$,

(b) $\Pr\{(X-3)^2 + (Y+2)^2 < 16\}$.

13. Ten cards are randomly dealt from the ordinary deck, and we are paid \$3 for each *spade* obtained, but we have to pay \$4 for each *face card* (J,Q,K). Find:

(a) The expected win (loss) in one such game, and the corresponding standard deviation.

(b) The probability of breaking even in one game.

(c) The expected total win (loss) in 500 independent rounds of this game, and the corresponding standard deviation.

14. A random variable X has a distribution with the following probability density function

$$f(x) = \begin{cases} 1 + x & -1 < x < 0 \\ 1 - x & 0 < x < 1 \end{cases}$$

Find the corresponding

(a) distribution function $F(x)$,

(b) quartile deviation,

(c) standard deviation,

(d) $\Pr(-\frac{1}{2} < X < \frac{1}{3})$.

15. One has to pay \$4 to play the following game: A die is rolled and the player receives \$12 for a six, \$6 for a five, \$3 for a four, and \$1.50 for a three (nothing for a one or two).

   (a) Find the mean and standard deviation of the net win in one round of this game.

   (b) Approximate the probability of winning money in 200 independent rounds of this game.

16. Assuming that X_1 and X_2 are independent random variables, both exponentially distributed with the mean of 1, find

$$\Pr(X_2 > X_1 + a)$$

   where $a > 0$.

17. Two random variables X and Y have the bivariate Normal distribution with $\mu_x = 3.7$, $\mu_y = 7.4$, $\sigma_x = 1.8$, $\sigma_y = 2.2$ and $\rho_{xy} = -0.91$ .Compute:

   (a) $\Pr(2X - 3Y < -16)$,

   (b) $\Pr(X > 4 \,|\, Y = 6.2)$.

18. Assuming that A, B, C, and D are independent events, having the probability of 0.69, 0.13, 0.81 and 0.28 respectively, find

$$\Pr\{((A \cap \overline{B}) \cup C) \cap (D \cup \overline{A})\}$$

19. There are 50 pieces of paper in a hat, each with a single digit (i.e. 0 to 9) written on it, 5 pieces for each digit. If ten of these are drawn, randomly and without replacement, find the probability of getting, exactly

   (a) two 5's, one 9, and no 0 (the rest arbitrary),

   (b) 3 singlets, 2 pairs, and one triplet (of identical digits).

20. There are three boxes, each containing 12 marbles of various colors. The number of marbles in Box 1, 2 and 3 is 4, 2 and 7, respectively. A box is chosen at random and 3 marbles are drawn from it (also randomly, and without replacement).

    (a) What is the probability that at least 2 of the drawn marbles will be red?

    (b) Assuming that the draw resulted in only one red marble, what is the conditional probability of having selected Box 3?

21. (a) Calculate the probability that at least 30 rolls of a die will be needed to get 5 sixes.

    (b) Approximate the probability that at least 3000 rolls of a die will be needed to get 500 sixes.

22. Consider a random independent sample of size 3 from an exponential distribution with the mean of 12. Find the probability that the sample mean will be bigger than 14.6 (the gamma distribution should help).

23. Consider the following joint pdf of X and Y:

$$f(x,y) = e^{-x} \qquad \text{when} \qquad x > y > 0 \qquad \text{(zero otherwise).}$$

    Find:

    (a) The marginal pdf of X.

    (b) The conditional pdf of Y given that $X = 1$.

    (c) $\Pr(X + Y < 2)$.

24. X_1, X_2 and X_3 are three independent random variables, each having the exponential distribution with the mean of 8.43.

    Compute $\Pr(X_1 + X_2 + X_3 > 25)$.

25. Consider a joint distribution of X and Y given by:

$$f(x,y) = \frac{c}{x+y} \qquad \begin{cases} x > 0 \\ y > 0 \\ x + y < 1 \end{cases}$$

    Find:

(a) The value of c.

(b) The marginal pdf of Y.

(c) The conditional pdf of X given that $Y = \frac{3}{4}$.

(d) $\Pr(X + 2Y < 1)$.

26. Consider a random independent sample of size 425 from the following distribution:

$X =$	-2	0	2	4
Pr:	0.46	0.34	0.15	0.05

. Using the Normal approximation compute $\Pr\left\{\sum_{i=1}^{425} X_i < -200\right\}$.

27. Let the random variables X and Y have the bivariate Normal distribution with $\mu_x = 1.24$, $\mu_y = 327$, $\sigma_x = 0.26$, $\sigma_y = 74$ and $\rho = -0.81$. Evaluate:

(a) $\Pr(Y > 400 \,|\, X = 0.96)$.

(b) $\Pr(300X - Y > 36)$.

28. X_1, X_2, X_3 and X_4 are four independent random variables each having the exponential distribution with the mean of 7.2.

(a) What is the moment generation function of $X_1 + X_2 + X_3 + X_4$?

(b) Compute $\Pr(20 < X_1 + X_2 + X_3 + X_4 < 30)$

(c) Also compute $\Pr\{\min(X_1, X_2, X_3, X_4) > 2\}$.

29. Customers arrive at the rate of 7.24 per hour. Using the Normal approximation, compute the probability of more than 50 customers arriving during the next 8 hours.

30. Let random variables Z_1 and Z_2 have the bivariate Normal distribution with $\rho = -0.94$ (both marginals are standardized Normal).

Evaluate:

(a) $\Pr(Z_1 > Z_2)$.

(b) $\Pr(Z_1 > 1.02 \,|\, Z_2 = -0.8)$.

31. Consider a random variable X with the following probability density function:

$$f(x) = \begin{cases} \frac{1}{3} & -1 < x < 0 \\ \frac{2}{3} & 0 < x < 1 \end{cases}$$

Find its mean, standard deviation and median. Also compute $\Pr\left(-\frac{2}{3} < X < \frac{1}{3}\right)$.

32. Let X and Y have a bivariate Normal distribution with $\mu_x = 15$, $\mu_y = -4$, $\sigma_x = 3$, $\sigma_y = 2$ and $\rho_{xy} = -\frac{7}{9}$. Compute:

    (a) $\Pr(X + Y > 10)$

    (b) $\Pr(X + Y > 10 \,|\, X = 13.4)$.

33. Consider two random variables X and Y with the following bivariate probability density function:

$$f(x, y) = c \cdot (x^2 + y^2) \qquad \text{when} \qquad x^2 + y^2 < 1 \quad \text{and} \quad 0 < y < x$$

(zero otherwise). Find:

    (a) the value of c,

    (b) $\mathbb{E}(X)$,

    (c) the conditional probability density function of X, given that $Y = \frac{1}{2}$.

34. Let X_1, X_2, X_3, X_4 and X_5 be a random independent sample of size 5 from the exponential distribution with $\beta = 7$. Compute:

    (a) $\Pr(\overline{X} < 6)$, where $\overline{X}$ is the corresponding sample mean,

    (b) $\Pr\{X_1 + X_3 + X_5 < 25 \cap \min(X_2, X_4) > 3\}$.

35. A certain random game has the following pay-off table (the values of X are in dollars):

$X =$	-3	6	9
Pr:	$\frac{7}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Using the normal approximation, calculate the probability of winning more than \$200 in 1000 independent rounds of this game (be careful with the continuity correction).