1.

$$\Pr\left[(A\cap\bar{B}\cap\bar{C})\cup(\bar{A}\cap B\cap C)\cup(A\cap\bar{B}\cap C)\right]=P(A\cap\bar{B}\cap\bar{C})+$$
$$P(\bar{A}\cap B\cap C)+P(A\cap\bar{B}\cap C)=P(A\cap\bar{B})-P(A\cap\bar{B}\cap C)+P(B\cap C)$$
$$-P(A\cap B\cap C)+P(A\cap C)-P(A\cap B\cap C)=P(A)-P(A\cap B)+$$
$$P(B\cap C)-P(A\cap B\cap C)=0.31-0.11+0.12-0.05=27\%$$

2.

$$\Pr\left[(A\cap\bar{B}\cap\bar{C}\cap D)\cup(\bar{A}\cap B\cap C)\cup(\bar{A}\cap B\cap D)\right]=$$
$$0.47\times0.79\times0.17\times0.55+0.53\times0.21\times0.83+$$
$$0.53\times0.21\times0.55-0.53\times0.21\times0.83\times0.55=13.75\%$$

3. A (B) represents blue (red) marbles being together.

$$\Pr(A\cap\bar{B})=\Pr(A)-\Pr(A\cap B)=\frac{3!\times7!-3!\times2\times6!}{9!}=5.952\%$$

4.

$Y\setminus X$	-2	-1	0	1	2	
1	c	c	c	c	c	$5c$
2	$\frac{c}{4}$	$\frac{c}{2}$	0	$2c$	$4c$	$\frac{27}{4}c$
3	$\frac{c}{9}$	0	0	0	$9c$	$\frac{82}{9}c$
	$\frac{49}{36}c$	$\frac{3}{2}c$	c	$3c$	$14c$	

(a) $c=\frac{1}{20+\frac{3}{4}+\frac{1}{9}}=\frac{36}{751}$

(b) $(-\frac{49}{18}-\frac{3}{2}+3+28)\times\frac{36}{751}=\frac{964}{751}=1.284$ and $(5+\frac{27}{2}+\frac{82}{3})\times\frac{36}{751}=\frac{1650}{751}=2.197$

(c) $(-2-1-\frac{2}{3}-1-1+1+4+2+16+54)\times\frac{36}{751}-\frac{820}{751}\times\frac{1650}{751}=1.0205$

(d) $(1+\frac{1}{2}+\frac{1}{3})\div(1+\frac{1}{4}+\frac{1}{9})=1.347$

5. (a) $\int_0^2 x^2dx+\int_2^3(3-x)dx=\frac{19}{6}\Rightarrow c=\frac{6}{19}$

(b)

$$\begin{cases} 0 & x\le 0\\ \frac{2}{19}x^3 & 0\le x<2\\ 1-\frac{3}{19}(3-x)^2 & 2\le x<3\\ 1 & x>3\end{cases}$$

(c) $1-\frac{3}{19}\times\frac{1}{4}-\frac{2}{19}=85.53\%$

(d) $\frac{6}{19}\int_0^2 x^3dx+\frac{6}{19}\int_2^3 x\cdot(3-x)dx=\frac{31}{19}=1.632$ and $\frac{6}{19}\int_0^2 x^4dx+\frac{6}{19}\int_2^3 x^2\cdot(3-x)dx-(\frac{31}{19})^2=0.2274$

6. (a) $\int_0^{\pi/2}\int_0^1 r^2\cos\theta drd\theta=\frac{1}{3}\Rightarrow c=3$

(b) $1-3\int_0^{1/2}\int_0^{1/2-y} x dx dy = \frac{15}{16}$

(c) $3\int_0^{\pi/2}\int_0^1 r^3 \sin^2\theta dr d\theta = 0.58905$

(d) $\frac{\int_0^{\sqrt{3}/2} x^2 dx}{\int_0^{\sqrt{3}/2} x dx} = 0.57735$

7.

$$W = 20X + 10Y + U - 21$$

$$\mathbb{E}(W) = 20\times 8\times \tfrac{1}{13} + 10\times 8\times \tfrac{1}{13} + 8\times \tfrac{1}{4} - 21 = -0.5385$$

$$\begin{aligned}\mathrm{Var}(W) &= \left(400\cdot\tfrac{1}{13}\cdot\tfrac{12}{13} + 100\cdot\tfrac{1}{13}\cdot\tfrac{12}{13} + \tfrac{1}{4}\cdot\tfrac{3}{4} - 400\cdot\tfrac{1}{13}\cdot\tfrac{1}{13}\right)\cdot 8\cdot\tfrac{44}{51} = 230\\ \sigma_w &= \sqrt{230} = 15.17\end{aligned}$$

$$\Pr(\text{breaking even}) = \frac{\binom{33}{7} + 3\cdot\binom{33}{6} + 3\cdot 11\cdot\binom{33}{6} + 3\cdot 11\cdot\binom{33}{5}}{\binom{52}{8}} = 6.907\%$$

8. (a)

$$\frac{1}{2}\left(1 - \frac{5^{12}}{6^{12}} - 12\cdot\frac{5^{11}}{6^{12}} - 66\cdot\frac{5^{10}}{6^{12}} + 1 - \frac{3^{12}}{4^{12}} - 12\cdot\frac{3^{11}}{4^{12}} - 66\cdot\frac{3^{10}}{4^{12}}\right) = 46.595\%$$

(b)

$$\frac{1}{2}\cdot\frac{12!}{2^6}\left(\frac{1}{6^{12}} + \frac{1}{8^8}\cdot\frac{1}{4^4}\right) = 0.2590\%$$

(c)

$$\frac{\frac{1}{6^{12}}}{\frac{1}{6^{12}} + \frac{1}{8^8}\cdot\frac{1}{4^4}} = 66.365\%$$

9. (a)

$$\frac{1}{3\cdot 5}\int_0^\infty\int_y^\infty \exp(-\frac{x}{3} - \frac{y}{5}) dx dy = \frac{3}{8}$$

(b)

$$\begin{aligned}5 - 3\times 3 + 2 &= -2\\ \sqrt{25 + 9\times 9} &= 10.296\end{aligned}$$

(c)

$$\mathrm{Cov}(Y - 3X + 2, 5X - 2Y + 1) = -15\cdot 9 - 2\cdot 25 = -185$$

10. (a)

$$1 - \exp(-\frac{10}{27}) = 30.95\%$$

(b)

$$\left(1 + \frac{50}{27} + \frac{50^2}{2\cdot 27^2}\right)\exp(-\frac{50}{27}) - \left(1 + \frac{80}{27} + \frac{80^2}{2\cdot 27^2}\right)\exp(-\frac{80}{27}) = 28.52\%$$

(c)

$$1-\left(1+\frac{100}{27}+\frac{100^2}{2\cdot 27^2}+\frac{100^3}{6\cdot 27^3}+\frac{100^4}{24\cdot 27^4}+\frac{100^5}{120\cdot 27^5}\right)\exp(-\frac{100}{27})=17.04\%$$

11. (a)

$$\frac{e^t}{(1-2t)^3}=1+7t+\frac{61}{2}t^2+O\left(t^3\right)$$

$\mu=7$ and $\sigma^2=61-7^2=12$

(b)

$$\frac{e^{3t}\cdot e^{-2t}}{(1+4t)^3}=\frac{e^t}{(1+4t)^3}$$

(c) $\mu_{\bar{X}}=20\times 7=140$, $\sigma^2_{\bar{X}}=\frac{12}{20}=\frac{3}{5}$

(d)

$$\frac{e^t}{(1-\frac{t}{10})^{60}}$$