1.

$$\Pr\left[(A \cup \bar{B} \cup C) \cap (\bar{A} \cup B \cup \bar{C})\right] = 1 - \Pr\left[(\bar{A} \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C)\right]$$

$$1 - \Pr(\bar{A} \cap B \cap \bar{C}) - \Pr(A \cap \bar{B} \cap C) = 1 - \Pr(\bar{A} \cap B) + \Pr(\bar{A} \cap B \cap C) - \Pr(A \cap C) + \Pr(A \cap B \cap C)$$

$$= 1 - \Pr(B) + \Pr(A \cap B) + \Pr(B \cap C) - \Pr(A \cap B \cap C) - \Pr(A \cap C) + \Pr(A \cap B \cap C)$$

$$= 1 - \Pr(B) + \Pr(A \cap B) + \Pr(B \cap C) - \Pr(A \cap C) = 1 - 0.54 + 0.21 + 0.30 - 0.25 = 72\%$$

2.

$$\Pr\left[\bar{A} \cap (B \cup C) \cap (C \cup D)\right] = \Pr(\bar{A}) \times \Pr[(B \cup C) \cap (C \cup D)]$$

$$= \Pr(\bar{A}) \times \Pr[(B \cap C) \cup C \cup (B \cap D) \cup (C \cap D)]$$

$$= \Pr(\bar{A}) \times \Pr[C \cup (B \cap D)]$$

$$= \Pr(\bar{A}) \times [\Pr(C) + \Pr(B)\Pr(D) - \Pr(B)\Pr(C)\Pr(D)]$$

$$= 0.58 \times (0.81 + 0.23 \times 0.54 - 0.23 \times 0.81 \times 0.54) = 48.355\%$$

3. ...

a.

$$P_4^8 \times \frac{7!}{11!} = 21.21\%$$

b.

$$4 \times \frac{4!8!}{11!} = 9.697\%$$

c.

$$4.5 \times \frac{4!8!}{11!} = 10.91\%$$

4.

s, $\geq$2	$\frac{\binom{13}{5}4^5}{\binom{52}{5}} \times 0 = 0$
d, $\geq$2	$\frac{\binom{13}{3}\times 10\times 4^3\times\binom{4}{2}+\binom{13}{2}\times 11\times 4\times\binom{4}{2}^2}{\binom{52}{5}} \times \frac{1}{4} = \frac{979}{8330}$
t, $\geq$2	$\frac{\binom{13}{2}\times 11\times 4^2\times\binom{4}{3}+13\times 12\times\binom{4}{3}\times\binom{4}{2}}{\binom{52}{5}} \times \frac{4}{8} = \frac{47}{4165}$
q, $\geq$2	$\frac{13\times 12\times 4}{\binom{52}{5}} \times \frac{11}{16} = \frac{11}{66\,640}$

a.

$$\frac{979}{8330} + \frac{47}{4165} + \frac{11}{66\,640} = 12.90\%$$

b.

$$\frac{\frac{47}{4165}}{\frac{979}{8330} + \frac{47}{4165} + \frac{11}{66\,640}} = 8.749\%$$

5. ..

a.

$$c = \frac{45}{164}$$

b.

$$\begin{aligned}\text{Var}(X) &= 0.7798 \\ \text{Cov}(X, Y) &= 0.3111\end{aligned}$$

c. Since $\mathbb{E}(X \mid Y = 0) = -\frac{9}{17}$, we get

$$-\frac{3 \times 9}{17} + 2 = \frac{7}{17} = 0.4118$$

6. ...

a. For X

$$\begin{aligned}\mu &= 9.3 \times \frac{17}{60} = 2.635 \\ \sigma &= \sqrt{9.3 \times \frac{17}{60}} = 1.623\end{aligned}$$

and Y

$$\begin{aligned}\mu &= 6 \times \frac{60}{9.3} = 38.71 \text{ min} \\ \sigma &= \sqrt{6} \times \frac{60}{9.3} = 15.80 \text{ min}\end{aligned}$$

b.

$$\begin{aligned}\Pr(X &> 5) = 1 - \exp\left(-9.3 \times \frac{17}{60}\right) \sum_{i=0}^{5} \frac{(9.3 \times \frac{17}{60})^i}{i!} = 5.165\% \\ \Pr(Y &> 25) = \left(\frac{9.3}{60}\right)^6 /5! \int_{25}^{\infty} y^5 \exp\left(-\frac{9.3y}{60}\right) dy = 80.435\%\end{aligned}$$

c.

$$\begin{aligned}P_{2x-3}(z) &= z^{-3} \exp\left(9.3 \times \frac{17}{60}(z^2 - 1)\right) \\ M_{3-2y}(t) &= \frac{e^{3t}}{(1 + 2 \cdot \frac{60}{9.3} t)^6}\end{aligned}$$

7. ..

a. Clearly

X_1	0	1
Pr	$\frac{10}{11}$	$\frac{1}{11}$

$\mathbb{E}(X_1) = \frac{1}{11}$, $\mathrm{Var}(X_1) = \frac{10}{121}$.

$\downarrow X_2 \ \ X_1 \rightarrow$	0	1
0	$\frac{91}{110}$	$\frac{1}{11} \cdot \frac{9}{10}$
1	$\frac{1}{11} \cdot \frac{9}{10}$	$\frac{1}{11} \cdot \frac{1}{10}$

implying that $\mathrm{Cov}(X_1, X_2) = \frac{1}{110} - \frac{1}{11} \cdot \frac{1}{11} = \frac{1}{1210}$. Thus

$$\begin{aligned} \mathbb{E}(X_{\mathrm{T}}) &= \frac{11}{11} = 1 \\ \mathrm{Var}(X_{\mathrm{T}}) &= 11 \times \frac{10}{121} + 2\binom{11}{2}\frac{1}{1210} = 1 \end{aligned}$$

b.

$$\begin{aligned} 1 &- \left(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} - \frac{1}{9!} + \frac{1}{10!} - \frac{1}{11!}\right) \\ &- \left(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} - \frac{1}{9!} + \frac{1}{10!}\right) = 26.42\% \end{aligned}$$

8.

$$P'(z) = \frac{e^{z-1}}{(3 - 2e^{z-1})^{3/2}}$$

implies that $\mu = 1$.

$$P''(z) = \frac{e^{z-1}(e^{z-1} + 3)}{(3 - 2e^{z-1})^{5/2}}$$

implies that $\sigma = \sqrt{4 + 1 - 1^2} = 2$. From the Taylor expansion of $P(z)$ we get: $\Pr(2 < X < 6) = 10.50\%$.

$$P_{3X+2}(z) = \frac{z^2}{\sqrt{3 - 2\exp(z^3 - 1)}}$$

Finally, $\Pr(14 < \sum_{i=1}^{7} X_i < 42) = 9.182\%$, from the expansion of $P(z)^7$.

9. ..

a.

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 - 2x + 3)^2} = \frac{1}{8}\pi\sqrt{2}$$

implying $c = \frac{8}{\pi\sqrt{2}} = 1.8006$.

b.

$$\begin{aligned} \mu &= \frac{8}{\pi\sqrt{2}} \int_{-\infty}^{\infty} \frac{x\, dx}{(x^2 - 2x + 3)^2} = 1 \\ \sigma &= \sqrt{\frac{8}{\pi\sqrt{2}} \int_{-\infty}^{\infty} \frac{(x-1)^2\, dx}{(x^2 - 2x + 3)^2}} = \sqrt{2} = 1.414 \end{aligned}$$

median is also equal to 1 (symmetry, or through F).

c.

$$\Pr(0.9 < \bar{X} < 1.08) \simeq \frac{1}{\sqrt{2\pi \cdot 2/50}} \int_{0.9}^{1.08} \exp(-\frac{(u-1)^2}{2 \cdot 2/50}) du = 34.69\%$$

10.

$$\begin{aligned} W &= 5X + 3Y - 20 \\ P_w(z) &= \sum_{i=0}^{7} \sum_{j=0}^{7-i} \frac{\binom{9}{i}\binom{12}{j}\binom{8}{7-i-j}}{\binom{29}{7}} z^{5i+3j-20} \end{aligned}$$

a. Adding coefficients of powers of z from 10 to 15 yields 0.6428% (one can do this directly, adding only 3 terms).

b. Expanding $P_w(z)^{10}$ and adding coefficients of powers of z from 10 to 150 yields 16.33%.

c. Since $\mu_w = -\frac{13}{29}$ and $\sigma_w^2 = \frac{17028}{841}$

$$\Pr(X_{\mathrm{T}} > 9.5) \simeq \frac{1}{\sqrt{2\pi \times 17028/841}} \int_{9.5}^{\infty} \exp\left(-\frac{(u+1300/29)^2}{2 \times 1702800/841}\right) du = 11.36\%$$

11. ..

a.

$$\begin{aligned} \mu_y &= 2 \times 2.3 - 3 \times 5.1 + 4 - 7 = -13.7 \\ \sigma_y^2 &= 2^2 \times 2.3^2 + 3^2 \times 5.1^2 + 4^2 = 271.25 \end{aligned}$$

b.

$$8 \times 2.3^2 - 6 \times 5.1^2 - 4^2 = -129.74$$

c.

$$1 - \exp\left(-\frac{4}{2.3}\right) \exp\left(-\frac{4}{5.1}\right) \exp\left(-\frac{4}{4}\right) = 97.05$$

d.

$$1 - \left(1 - \exp(-\frac{6}{2.3})\right)\left(1 - \exp(-\frac{6}{5.1})\right)\left(1 - \exp(-\frac{6}{4})\right) = 50.225\%$$