

MATH 2F81 FIRST MIDTERM OCTOBER 8, 2010
Full credit given for 5 correct and complete answers.
Four significant digits required for numerical answers, e.g. 0.0005824
One sheet of notes and a Maple workspace allowed. Duration: 2 hours

1. Three married couples and 10 other people (5 males and 5 females) are randomly seated at a round table. What is the probability that
 - (a) at least one husband will sit next to his wife,
 - (b) the males and females alternate (regardless of who is married to whom).
2. When 12 dice are rolled, what is the probability of getting (exactly):
 - (a) three sixes,
 - (b) three sixes and two fives,
 - (c) two quadruplets (of identical values).
3. Consider all $11!$ permutations of the letters p r o b l e m a t i c. In how many of these permutations are
 - (a) all 11 letters misplaced,
 - (b) exactly 7 letters misplaced (the remaining 4 remaining in their original positions).
4. Given that

$$\begin{aligned}\Pr(A) &= 0.31, & \Pr(B) &= 0.33, & \Pr(C) &= 0.37 \\ \Pr(A \cap B) &= 0.11, & \Pr(A \cap C) &= 0.13, & \Pr(B \cap C) &= 0.12 \\ & & \text{and } \Pr(A \cap B \cap C) &= 0.05\end{aligned}$$

find

$$\Pr [(A \cup \bar{B} \cup C) \cap (A \cup \bar{B} \cup \bar{C})]$$

Hint: One way to proceed is to compute the probability of the complement first.

5. Five cards are dealt to each of two players from a well shuffled deck of 52 cards. What is the probability that:
 - (a) the first player gets (exactly) one jack and one spade,
 - (b) each player gets (exactly) one jack and one spade.

6. Five cards are dealt to each of two players from a well shuffled deck of 52 cards. What is the probability that the first player gets more aces than the second player. Hint: Computing the probability of both getting the *same* number of aces may lead to the answer more easily.

7. Consider the following random experiment. Five cards are dealt from a regular deck. Then, a coin is flipped as many times as the number of aces obtained.
 - (a) What is the probability of getting more than 1 heads?
 - (b) Given that we have gotten more than one heads, what is the conditional probability of having obtained at least 3 aces?