

MATH 2F81 FIRST MIDTERM OCTOBER 7, 2011
Full credit given for answering, correctly, 5 (out of 7) questions.
Four significant digits required for numerical answers, e.g. 0.0005824
One sheet of notes and a Maple workspace allowed. Duration: 2 hours

1. Four married couples and 7 single men are randomly seated at a round table. What is the probability that
- (a) no two females sit next to each other,
 - (b) no husband and wife sit next to each other? Hint: Introduce A (first couple sit together), B (second couple sit together), ..., and use the power of Boolean algebra.

2. When

$$\left(1 - \frac{x}{20} + x^2\right)^{6000}$$

gets expanded and terms with like powers collected, what (and why - give the details) is the coefficient of

- (a) x^5 ,
 - (b) x^{11995} ?
3. Three friends and 33 other boys are randomly divided into six teams of 6 players each. What is the probability that
- (a) all three friends end up on the same team,
 - (b) they get separated, each playing for different team?

4. Given that

$$\begin{aligned}\Pr(A) &= 0.31, & \Pr(B) &= 0.33, & \Pr(C) &= 0.37 \\ \Pr(A \cap B) &= 0.11, & \Pr(A \cap C) &= 0.13, & \Pr(B \cap C) &= 0.12 \\ & & & & \text{and } \Pr(A \cap B \cap C) &= 0.05\end{aligned}$$

find

- (a)

$$\Pr [(A \cap \bar{B}) \cup (\bar{B} \cap C) \cup (A \cap C)]$$

(b)

$$\Pr [(A \cup B) \cap (B \cup C) \cap (A \cup C)]$$

5. Six cards are dealt from a well shuffled deck of 52 cards. What is the probability of getting (exactly)
- (a) 1 ace, 2 spades, and 2 diamonds,
 - (b) exactly one 'triplet' (such as 3 sevens plus 3 other cards which are not sevens)?
6. Five cards are dealt to each of four players from a well shuffled deck of 52 cards. What is the probability that
- (a) each of them gets exactly 1 spade,
 - (b) the third and fourth player get exactly 1 ace each (regardless of what the other two players get)?
7. Consider randomly permuting the letters p r o b l e m s.
- (a) How many of all possible $8!$ permutations have *all* eight letters misplaced?
 - (b) What is the probability that at least 3 of the letters will be placed in their original location?