MATH 2F81 FIRST MIDTERM OCTOBER 10, 2012
Full credit given for 4 correct and complete answers.
Four significant digits required for numerical answers, e.g. 5.270×10^{-5}
One sheet of notes and a Maple workspace allowed. Duration: 1.5 hours

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1. Three married couples and 5 unrelated ladies (11 people in total) are randomly seated in a *row*. What is the probability that

   (a) none of the 3 husbands sits next to his wife,

   (b) no two husbands sit next to each other. Hint: Let A_{12} (A_{13} and A_{23}) mean 'husband 1 and 2 (1 and 3, 2 and 3) sit next to each other'. We need $\Pr(\overline{A_{12}} \cap \overline{A_{13}} \cap \overline{A_{23}})$....

2. 8 cards are randomly dealt from the standard deck of 52 cards. What is the probability of getting a hand with exactly

   (a) 2 aces, 2 spades and 2 diamonds,

   (b) exactly two pairs (and anything else, as long as it is not another pair).

3. Three friends (Jim, Joe and Tom) and another 9 boys are randomly divided into 3 teams of 4 players. What is the probability that

   (a) Jim and Joe, but not Tom, will be on the same team,

   (b) at least two of the three friends will play for the same team.

4. Consider all 5040 permutations of the word *special*. How many of them

   (a) have more than 4 of the original letters misplaced,

   (b) start *and* end with a vowel (a, i, e)?

5. Given that

$$\begin{aligned} \Pr(A) &= 0.30, \quad \Pr(B) = 0.32, \quad \Pr(C) = 0.35 \\ \Pr(A \cap B) &= 0.09, \quad \Pr(A \cap C) = 0.13, \quad \Pr(B \cap C) = 0.12 \\ &\quad \text{and } \Pr(A \cap B \cap C) = 0.04 \end{aligned}$$

   find

(a)

$$\Pr\left[\bar{A} \cup \bar{B} \cup C\right]$$

(b)

$$\Pr\left[(\bar{A} \cap \bar{B} \cap C) \cup \bar{C}\right]$$

6. Consider expanding

$$\left(3 + 2x^2y - 4t^2x + y^2\right)^{29}$$

   (a) How many terms are there in this expansion?

   (b) One of these terms will have the form of $C \cdot x^{17}y^{21}t^{14}$, where C is a number. Find C using the appropriate formula.

7. Consider the following game: a die is rolled followed by randomly dealing as many cards (from a standard deck) as the number of dots obtained.

   (a) What is the probability of getting at least one spade?

   (b) Given that at least one spade has been dealt (this is the only piece of information we get about the outcome), what is the conditional probability that fewer than 3 dots showed on the die?