1. (a) Using A, B and C for the first, second and third couple (respectively) sitting next to each other, we get

$$\Pr(A \cup B \cup C) = 3\Pr(A) - 3\Pr(A \cap B) + \Pr(A \cap B \cap C)$$
$$= 3\frac{2 \cdot 15!}{16!} - 3\frac{2^2 \cdot 14!}{16!} + \frac{2^3 \cdot 13!}{16!} = 32.74\%$$

(b)

$$2\frac{8! \cdot 8!}{16!} = 0.01554\%$$

2.

(a)

$$\frac{\frac{\binom{18}{3,5,5,5}}{3!} - \frac{\binom{16}{3,3,5,5}}{2!} - \frac{\binom{16}{1,5,5,5}}{3!}}{\frac{\binom{20}{5,5,5,5}}{4!}} = \frac{\frac{\binom{18}{3}\binom{15}{5}\binom{10}{5}}{3!} - \frac{\binom{16}{3}\binom{13}{3}\binom{10}{5}}{2!} - \frac{16\binom{15}{5}\binom{10}{5}}{3!}}{\frac{\binom{20}{5}\binom{15}{5}\binom{10}{5}}{4!}} = 16.51\%$$

(b)

$$\frac{4\frac{\binom{16}{2,4,5,5}}{2!} + \frac{\binom{16}{1,5,5,5}}{3!}}{\frac{\binom{20}{5,5,5,5}}{4!}} = \frac{4\frac{\binom{16}{2}\binom{14}{4}\binom{10}{5}}{2!} + \frac{16\binom{15}{5}\binom{10}{5}}{3!}}{\frac{\binom{20}{5}\binom{15}{5}\binom{10}{5}}{4!}} = 12.80\%$$

3.

$$\binom{200}{2,1,197}\left(\frac{1}{3}\right)^2\left(\frac{1}{5}\right)\left(\frac{14}{15}\right)^{197} + \binom{200}{2,2,196}\left(\frac{4}{5}\right)^{196}\left(\frac{1}{3}\right)^2\left(\frac{-1}{4}\right)^2\left(\frac{14}{15}\right)^{196}$$
$$+\binom{200}{5}\left(\frac{1}{3}\right)^5\left(\frac{14}{15}\right)^{195} = 15.09$$

4.

$$\Pr\left[(A \cup \bar{B}) \cap \overline{B \cap C} \cap (A \cup \bar{C})\right]$$
$$= 1 - \Pr\left[(\bar{A} \cap B) \cup (B \cap C) \cup (\bar{A} \cap C)\right]$$
$$= 1 - \Pr(\bar{A} \cap B) - \Pr(\bar{A} \cap C) - \Pr(B \cap C) + 3\Pr(\bar{A} \cap B \cap C) - \Pr(\bar{A} \cap B \cap C)$$
$$= 1 - \Pr(B) + \Pr(A \cap B) - \Pr(C) + \Pr(A \cap C) - \Pr(B \cap C) + 2\Pr(B \cap C) - 2\Pr(A \cap B \cap C)$$
$$= 1 - 0.33 + 0.11 - 0.37 + 0.13 + 0.12 - 2 \times 0.05 = 0.56$$

(using Venn diagram is also OK).

5. Break according to: one player getting the jack of spades + no player getting the jack of spades

$$3 \times \frac{\binom{36}{4} \times 3 \cdot 12\binom{32}{3} \times 2 \cdot 11\binom{29}{3}}{\binom{52}{5}\binom{47}{5}\binom{42}{5}} + \frac{3 \times 12 \times \binom{36}{3} \times 2 \cdot 11\binom{33}{3} \times 10\binom{30}{3}}{\binom{52}{5}\binom{47}{5}\binom{42}{5}} = 0.1117\%$$

6. ....

(a) Same number of spades as diamonds:

$$\frac{\binom{26}{5} + 13 \cdot 13\binom{26}{3} + \binom{13}{2} \cdot \binom{13}{2} \cdot 26}{\binom{52}{5}} = \frac{12,757}{49,980}$$

Answer:

$$\frac{1 - \frac{12\,757}{49\,980}}{2} = 37.24\%$$

(b)

$$\frac{13 \cdot 12 \cdot \binom{4}{2}\binom{4}{3}}{\binom{52}{5}} = 0.1441\%$$

7. This can be made into a probability tree

0, $\geq$1	$\frac{1}{16} \cdot 0$	○	
0, E	$\frac{1}{16} \cdot 1$		
1, $\geq$1	$\frac{4}{16} \cdot \frac{1}{4}$	○	
1, E	$\frac{4}{16} \cdot \frac{3}{4}$		
2, $\geq$1	$\frac{6}{16} \cdot \left(1 - \frac{\binom{39}{2}}{\binom{52}{2}}\right)$	○	
2, E	$\frac{6}{16} \cdot \frac{\binom{39}{2}}{\binom{52}{2}}$		
3, $\geq$1	$\frac{4}{16} \cdot \left(1 - \frac{\binom{39}{3}}{\binom{52}{3}}\right)$	○	✓
3, E	$\frac{4}{16} \cdot \frac{\binom{39}{3}}{\binom{52}{3}}$		
4, $\geq$1	$\frac{1}{16} \cdot \left(1 - \frac{\binom{39}{4}}{\binom{52}{4}}\right)$	○	✓
4, E	$\frac{1}{16} \cdot \frac{\binom{39}{4}}{\binom{52}{4}}$		

(a)

$$\frac{4}{16}\cdot\frac{1}{4}+\frac{6}{16}\cdot\left(1 - \frac{\binom{39}{2}}{\binom{52}{2}}\right)+\frac{4}{16}\cdot\left(1 - \frac{\binom{39}{3}}{\binom{52}{3}}\right)+\frac{1}{16}\cdot\left(1 - \frac{\binom{39}{4}}{\binom{52}{4}}\right) = \frac{139,301}{333,200} = 41.81\%$$

(b)

$$\frac{\frac{4}{16} \cdot \left(1 - \frac{\binom{39}{3}}{\binom{52}{3}}\right) + \frac{1}{16} \cdot \left(1 - \frac{\binom{39}{4}}{\binom{52}{4}}\right)}{\frac{139301}{333200}} = 45.48\%$$