MATH 2F81   SECOND MIDTERM   NOVEMBER 19, 2009

Full credit given for 3 correct and complete answers.

One sheet of notes and Maple workspace permitted.

Duration: 1 hour.

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1. Cards are dealt from a well shuffled deck, one by one, until the first spade appears. If X is the number of cards dealt (including the spade), and i is a positive integer,

   (a) find a formula for $\Pr(X > i)$,

   (b) convert it into $\Pr(X = i)$; what is the range of possible values of i?

   (c) Compute the mean and standard deviation of X.

2. Consider playing the following game: Roll a die and *pay* \$2 for each dot, then randomly draw 9 cards from a deck of 52 cards and *be paid* \$3 for each heart. Find:

   (a) The expected value and standard deviation of your net win.

   (b) Probability of winning (net) more than \$6 in a single game. Hint: Use probability generating function.

   (c) Probability of winning (net) more than \$6 (in total) in 5 rounds of this game.

3. Given the following joint probability function

$$f_{xy}(i, j) = c \cdot (i^2 + j^2)$$

   of two random variables X and Y, where i and j are two integers such that

$$-1 \leq i \leq 2$$
$$\max(1, i) \leq j \leq 2 + |i|$$

   find:

   (a) The value of c.

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(b) $\mathrm{Cov}(X, Y)$.

(c) $\mathbb{E}(3X - 1 \mid Y = 2)$.

4. Customers arrive at an average rate of 18.3 per hour. If the store opens at 8:00, find the probability

   (a) of the third customer arriving between 8:05 and 8:10,

   (b) of getting more than 6 customers between 8:42 and 9:11.

   (c) If X is the number of customers arriving between 8:42 and 9:11, find expected value and standard deviation of $2 - 3X$.

5. Roll 3 dice followed by dealing as many cards (from a well shuffled deck) as the number of sixes obtained. Find the distribution of the number of spades dealt, and the corresponding mean, standard deviation, skewness and kurtosis (expect the last two numbers to be unusually high).