1.

$$\Pr[(A \cup \bar{B}) \cap \overline{B \cup C} \cap (C \cup \bar{D})] = 1 - \Pr[(\bar{A} \cap B) \cup B \cup C \cup (\bar{C} \cap D)$$
$$= \quad 1 - \Pr[B \cup C \cup D)$$

as $\bar{A} \cap B$ is a subset of B, and $C \cup (\bar{C} \cap D) = \Omega \cap (C \cup D) = C \cup D$. Now

$$1 - \Pr[B \cup C \cup D) = \Pr(\bar{B} \cap \bar{C} \cap \bar{D}) = 0.79 \times 0.17 \times 0.45 = 6.0435\%$$

2.

$\frac{X\rightarrow}{Y\downarrow}$	0	1	2	3	4	
0	$1c$	$3c$	$5c$	$7c$	$9c$	25c
1	$2c$	$4c$	$6c$	0	0	12c
2	$3c$	0	0	0	0	3c
	6c	7c	11c	7c	9c	

a. $c = \frac{1}{40}$

b.

$$\mu_x = \frac{7+22+21+36}{40} = \frac{43}{20}$$
$$\mu_y = \frac{12+6}{40} = \frac{9}{20}$$
$$\mathrm{Var}(X) = \frac{7+44+63+144}{40} - (\frac{43}{20})^2 = \frac{731}{400}$$
$$\mathrm{Var}(Y) = \frac{12+12}{40} - (\frac{9}{20})^2 = \frac{159}{400}$$
$$\mathrm{Cov}(X,Y) = \frac{4+12}{40} - \frac{43}{20} \cdot \frac{9}{20} = -\frac{227}{400}$$

Answer:

$$\rho = \frac{-227}{\sqrt{731 \cdot 159}} = -0.6658$$

c.

Y|X=0	0	1	2
Pr	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

Answer:

$$2 \cdot (1-2) \cdot \frac{3}{6} = -1$$

3. ...

a.

$$\Lambda_1 = 12.3 \times \frac{6}{60} = 1.23$$
$$\Lambda_2 = 12.3 \times \frac{12}{60} = 2.46$$

Answer:

$$(1+1.23)e^{-1.23} - (1+2.46)e^{-2.46} = 35.62\%$$

b.

$$\Lambda = 12.3 \times \frac{26}{60} = 5.33$$

Answer:

$$e^{-5.33} \sum_{i=0}^{4} \frac{5.33^i}{i!} = 38.46\%$$

c.

$$\begin{aligned} 3 - 2 \times 5.33 &= -7.66 \\ 2 \times \sqrt{5.33} &= 4.617 \end{aligned}$$

4.

SS	R	BR	BBR	BBBR	BBBBR	BBBBBR
X	1	2	3	4	5	6
Pr	$\frac{7}{12}$	$\frac{5}{12} \cdot \frac{7}{11} = \frac{35}{132}$	$\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{7}{10} = \frac{7}{66}$	$\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} \cdot \frac{7}{9} = \frac{7}{198}$	$\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{7}{8} = \frac{7}{792}$	$\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = \frac{1}{792}$

$$\mu = \sum_{i=1}^{6} i \times \Pr(X = i) = \frac{13}{8} = 1.625$$

$$\text{Var}(X) = \sum_{i=1}^{6} (i - \frac{13}{8})^2 \times \Pr(X = i) = \frac{455}{576} = 0.7899$$

$$\frac{\sum_{i=1}^{6} (i - \frac{13}{8})^3 \times \Pr(X = i)}{\text{Var}(X)^{3/2}} = 1.519$$

$$\frac{\sum_{i=1}^{6} (i - \frac{13}{8})^4 \times \Pr(X = i)}{\text{Var}(X)^2} = 5.140$$

5. ...

a.

$$\sum_{i=0}^{3} \binom{19}{i} (\frac{1}{6})^i (\frac{5}{6})^{19-i} - \sum_{i=0}^{3} \binom{30}{i} (\frac{1}{6})^i (\frac{5}{6})^{30-i} = 36.74\%$$

b.

$$\begin{aligned} 4 \times 6 &= 24 \\ \sqrt{4 \times 6 \times 5} &= 10.95 \end{aligned}$$

c. We know that $P(z) = z^4(6 - 5z)^{-4}$. The third derivative is $144z(36 + 90z + 25z^2)(6 - 5z)^{-7}$, evaluated at $z = 1$ yields $21,744$.

6.

$$W = 3X + Y - 2Z$$

where X, Y and Z is the number of aces, face cards and spades, respectively.

$$\begin{aligned}\mathbb{E}\,(3X+Y-2Z) &= 3\cdot\frac{5}{13}+\frac{5\cdot 3}{13}-2\cdot\frac{5}{4}=-19.23\,¢\\ \mathrm{Var}(3X+Y-2Z) &= 5\left(9\cdot\frac{1}{13}\frac{12}{13}+\frac{3}{13}\frac{10}{13}+4\cdot\frac{1}{4}\frac{3}{4}-6\cdot\frac{1}{13}\frac{3}{13}\right)\frac{52-5}{52-1}=\frac{77\,315}{11\,492}\end{aligned}$$

implying that

$$\sigma_w=\sqrt{\frac{77\,315}{11\,492}}=2.594\ \$$$

Losing more than \$7 can happen only when we get 4 spades, no ace and no face card, of we get 5 spades, no ace and up to 2 face cards. The probability is:

$$\frac{\binom{9}{4}\binom{27}{1}+\binom{9}{5}+\binom{9}{4}\binom{3}{1}+\binom{9}{3}\binom{3}{2}}{\binom{52}{5}}=0.1600\%$$

7. ...

a. The first derivative of $P(z)$ is

$$\frac{80\cdot(3+2z)^3}{(7-2z)^5}$$

evaluated at $z=1$ yields the mean of $\frac{16}{5}=3.2$. The second derivative evaluated at $z=1$ yields $\frac{256}{25}$. This implies

$$\sigma=\sqrt{\frac{256}{25}+\frac{16}{5}-(\frac{16}{5})^2}=1.789$$

b. Expanding $P(z)$ in Taylor series, the coefficients of z^3 is: 23.20%

The expansion is of course done by Maple. Doing it 'by hand' takes a lot longer, but it is still feasible:

$$(3+2z)^4\cdot 7^{-4}\cdot(1-\tfrac{2}{7}z)^{-4}=$$

$$7^{-4}\cdot\left(3^4+4\cdot 3^3\cdot(2z)+6\cdot 3^2\cdot(2z)^2+4\cdot 3\cdot(2z)^3+...\right)\cdot$$

$$\left(1+\tbinom{-4}{1}\cdot\tfrac{-2}{7}z+\tbinom{-4}{2}\cdot(\tfrac{-2}{7}z)^2+\tbinom{-4}{3}\cdot(\tfrac{-2}{7}z)^3+...\right)$$

This implies that the answer is

$$7^{-4}\cdot\left(3^4\cdot\tbinom{-4}{3}\cdot(\tfrac{-2}{7})^3+4\cdot 3^3\cdot(2)\cdot\tbinom{-4}{2}\cdot(\tfrac{-2}{7})^2+6\cdot 3^2\cdot(2)^2\cdot\tbinom{-4}{1}\cdot\tfrac{-2}{7}+4\cdot 3\cdot(2)^3\right)$$

which evaluates to the same 23.20%. This is what students had to do in pre-Maple days!

c.

$$z^5\left(\frac{3+2z^2}{7-2z^2}\right)^4$$

....

This note is for the 'purists' who want to avoid using Maple entirely. This is how we can evaluate the third derivative of $z^4(6-5z)^{-4}$ at $z=1$, utilizing the following product rule:

$$(f \cdot g)''' = f''' \cdot g + 3f'' \cdot g' + 3f' \cdot g'' + f \cdot g'''$$

In our case, this yields

$$4 \cdot 3 \cdot 2 + 3 \cdot 4 \cdot 3 \cdot (-4) \cdot (-5) + 3 \cdot 4 \cdot (-4) \cdot (-5) \cdot (-5)^2 + (-4) \cdot (-5) \cdot (-6) \cdot (-5)^3 = 21,744$$

Are not we lucky to have Maple!