Full credit given for 6 correct and complete answers (out of 8)
One sheet of notes permitted   Duration: 2 hours

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1. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.47$, $\Pr(B) = 0.21$, $\Pr(C) = 0.83$ and $\Pr(D) = 0.55$, find

$$\Pr[(A \cup \bar{C}) \cap (\bar{B} \cup D) \cap \bar{D}]$$

2. Consider rolling two regular tetrahedrons (dice with four sides, marked 1, 2, 3 and 4). Let X be the smaller of the two numbers obtained, and Y be the larger number (when two identical numbers are obtained, X and Y will be both assigned this number). Compute the expected value of each X and Y. Also: find the covariance between X and Y.

3. You pay \$5 to play the following game: 3 regular dice are rolled and you lose your \$5 if no six appears, get your \$5 back if 1 six appears, get \$15 (\$5 back plus \$10 extra) if 2 sixes appear, and get \$400 (\$5 plus \$395) for 3 sixes.

   (a) What is the expected value of your net win, and the corresponding standard deviation?

   (b) What is the probability that, in three rounds of such a game, you will break even (net win equals zero)?

4. A regular die is rolled, followed by flipping a coin as many times as the number of dots shown on the die.

   (a) Compute the probability of getting more heads than tails.
       Hint: for odd number of flips, the probability of more heads than tails is $\frac{1}{2}$, for even number of flips, this probability equals $\frac{1 - \Pr(\text{same } \# \text{ of heads as tails})}{2}$.

   (b) Given that more heads than tails were obtained, what is the conditional probability that the die roll resulted in more than 3 dots?

5. Assuming that X has the following distribution:

$X =$	-1	0	1	2	4
Pr	0.12	0.23	0.17	0.21	0.27

find the expected value and standard deviation of $Y \equiv X^2$.

6. We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Based on this, prove that

$$P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$$

7. Consider randomly dealing five cards to each of four players. If the first player gets exactly 1 ace, what is the conditional probabillity that none of the other three players gets more than 1 ace?

8. Let the joint probability function of X and Y be given by:

$$f_{XY}(i, j) = c(i + j^2 + ij) \quad \text{for} \quad 0 \le i \le 3 \quad \text{and} \quad i \le j \le 3$$

where c is the appropriate constant.

   (a) Find the marginal distribution of Y.
   (b) Find the conditional distribution of X given that $Y = 2$.