MATH 2F81 SECOND MIDTERM NOVEMBER 12, 2007
Full credit given for 3 correct and complete answers
One sheet of notes and Maple workspace permitted. **Duration: 1 hour**

1. Let X and Y be two integer-valued random variables with the joint probability function given by

$$f(i,j) = c \cdot i \cdot j$$

where i and j are any two *positive* integers such that $i + j \leq 4$ (otherwise, $f(i,j) = 0$).

   (a) Find the value of c. Are X and Y independent?

   (b) If $U = 3X - 5$, find the moment generating function of U.

   (c) Compute $\mathrm{Cov}(X, Y)$.

   (d) Compute $\mathbb{E}(\frac{1}{X} \mid Y = 1)$.

2. Suppose a die has 1 dot on three of its faces, 2 dots on two faces, and 3 dots on one face. You roll this die once and, according to the number of dots obtained, you deal that many cards from a well shuffled deck. Find

   (a) the distribution of the number of spades dealt,

   (b) the conditional probability of having obtained 1 dot on the die, given that one spade was dealt.

3. Consider a game with the following payoff table (in dollars):

X	-3	0	8
Pr	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

   Suppose you play 3 rounds of this game. Find

   (a) the expected value and standard deviation of the *total* amount you win,

   (b) the probability of winning (total, net) at least \$15.

   (c) If you play 20 rounds of this game, what is the probability that you lose \$3 in more than 10 of these rounds.

4. Students enter the library (randomly and idividually) at an average rate of 147 per hour. Let X be the number of students entering the library during the next 5 minutes. Also, let Y be the number of rolls of a regular die needed to get the third six. Compute:

   (a) $\Pr(X < 7)$,

   (b) $\Pr(Y < 20)$,

   (c) $\Pr(X + Y < 27)$. Hint: Assume that X and Y are independent, and use the corresponding probability generating function.

5. Consider paying \$10 to play to following game: First, the ace of clubs is removed from a deck of cards. Then, the remaining cards (51 of them) are shuffled and 6 cards are dealt. You get \$20 for each ace and \$5 for each spade thus obtained (\$25 for the ace of spades). Find

   (a) the probability of winning (net) exactly \$25,

   (b) the expected value and standard deviation of your net win.