1.

$$\Pr[(A\cup\bar{C})\cap(\bar{B}\cup D)\cap\bar{D}]=\Pr(A\cup\bar{C})\cdot\Pr[(\bar{B}\cup D)\cap\bar{D}]=$$
$$(0.47+0.17-0.47\times 0.17)\times\Pr(\bar{B}\cap D)=$$
$$0.5601\times 0.79\times 0.55=24.34\%$$

2.

| | | | |
|---|---|---|---|
| 1,1 | 1,2 | 1,3 | 1,4 |
| 1,2 | 2,2 | 2,3 | 2,4 |
| 1,3 | 2,3 | 3,3 | 3,4 |
| 1,4 | 2,4 | 3,4 | 4,4 |

(all equally likely) converts to the following joint distribution

| $Y\backslash X$ | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | $\frac{1}{16}$ | 0 | 0 | 0 | $\frac{1}{16}$ |
| 2 | $\frac{2}{16}$ | $\frac{1}{16}$ | 0 | 0 | $\frac{3}{16}$ |
| 3 | $\frac{2}{16}$ | $\frac{2}{16}$ | $\frac{1}{16}$ | 0 | $\frac{5}{16}$ |
| 4 | $\frac{2}{16}$ | $\frac{2}{16}$ | $\frac{2}{16}$ | $\frac{1}{16}$ | $\frac{7}{16}$ |
| | $\frac{7}{16}$ | $\frac{5}{16}$ | $\frac{3}{16}$ | $\frac{1}{16}$ | |

$\mathbb{E}(X)=\frac{7+10+9+4}{16}=\frac{15}{8}$. $\mathbb{E}(Y)=\frac{1+6+15+28}{16}=\frac{25}{8}$,
$\mathrm{Cov}(X,Y)=\frac{1+4+4+6+12+9+8+16+24+16}{16}-\frac{15}{8}\times\frac{25}{8}=\frac{25}{64}$

3.

| $W$ | $-5$ | 0 | 10 | 395 |
|---|---|---|---|---|
| Pr | $\frac{5^3}{6^3}$ | $3\frac{5^2}{6^3}$ | $3\frac{5}{6^3}$ | $\frac{1}{6^3}$ |

(a) $\mathbb{E}(X)=\frac{-5^4+150+395}{6^3}=-\frac{10}{27}$, $\sigma_X=\sqrt{\frac{5^5+1500+395^2}{6^3}-\frac{10^2}{27^2}}=27.269$

(b) $(3\frac{5^2}{6^3})^3+3\times 3\frac{5}{6^3}\times(\frac{5^3}{6^3})^2=11.16\%$

(a)

$$\Pr(A)=\Pr(A|B_1)\Pr(B_1)+\Pr(A|B_2)\Pr(B_2)+\Pr(A|B_3)\Pr(B_3)+$$
$$\Pr(A|B_4)\Pr(B_4)+\Pr(A|B_5)\Pr(B_5)+\Pr(A|B_6)\Pr(B_6)=$$
$$\frac{\frac{1}{2}+\frac{1-\frac{2}{4}}{2}+\frac{1}{2}+\frac{1-\frac{6}{16}}{2}+\frac{1}{2}+\frac{1-\frac{20}{64}}{2}}{6}=40.10\%$$

(b)

$$\frac{\frac{1-\frac{6}{16}}{2}+\frac{1}{2}+\frac{1-\frac{20}{64}}{2}}{\frac{1}{2}+\frac{1-\frac{2}{4}}{2}+\frac{1}{2}+\frac{1-\frac{6}{16}}{2}+\frac{1}{2}+\frac{1-\frac{20}{64}}{2}}=48.05\%$$

4.

$$E(Y)=0.12+0.17+4\times 0.21+16\times 0.27=5.45$$
$$E(Y^2)=0.12+0.17+16\times 0.21+16^2\times 0.27=72.77$$
$$\sigma_Y=\sqrt{72.77-5.45^2}=6.563$$

5. We need to prove:

$$\frac{\Pr[(A \cup B) \cap C)}{\Pr(C)} = \frac{\Pr(A \cap C)}{\Pr(C)} + \frac{\Pr(B \cap C)}{\Pr(C)} - \frac{\Pr(A \cap B \cap C)}{\Pr(C)}$$

or

$$\Pr[(A \cup B) \cap C) = \Pr(A \cap C) + \Pr(B \cap C) - \Pr(A \cap B \cap C)$$

Starting from LHS:

$$\begin{aligned}&\Pr[(A \cup B) \cap C) = \Pr[(A \cap C) \cup (B \cap C)] = \\ &\Pr(A \cap C) + \Pr(B \cap C) - \Pr(A \cap C \cap B \cap C) = \\ &\Pr(A \cap C) + \Pr(B \cap C) - \Pr(A \cap B \cap C)\end{aligned}$$

6. Let $B$ ($C$ and $D$) means Mr. B (C and D) getting more than 1 ace; $A$ represents Mr. A (the first player) getting 1 ace. We need

$$\begin{aligned}&P(\bar{B} \cap \bar{C} \cap \bar{D}|A) = \Pr(\overline{B \cup C \cup D}|A) = \\ &1 - \Pr(B \cup C \cup D|A) = \\ &1 - \Pr(B|A) - \Pr(C|A) - P(D|A) = \\ &1 - 3\frac{\binom{3}{2}\binom{44}{3} + \binom{3}{3}\binom{44}{2}}{\binom{47}{5}} = 92.04\%\end{aligned}$$

7.

| $Y\backslash X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | $0+0+0$ | 0 | 0 | 0 |
| 1 | $(0+1+0)c$ | $(1+1+1)c$ | 0 | 0 |
| 2 | $(0+4+0)c$ | $(1+4+2)c$ | $(2+4+4)c$ | 0 |
| 3 | $(0+9+0)c$ | $(1+9+3)c$ | $(2+9+6)c$ | $(3+9+9)c$ |

or

(a)

| $Y\backslash X$ | 0 | 1 | 2 | 3 | |
|---|---|---|---|---|---|
| 1 | $c$ | $3c$ | 0 | 0 | $\frac{4}{85}$ |
| 2 | $4c$ | $7c$ | $10c$ | 0 | $\frac{21}{85}$ |
| 3 | $9c$ | $13c$ | $17c$ | $21c$ | $\frac{60}{85}$ |

which implies that $c = \frac{1}{85}$.

(b)

| $X\|Y=2$ | 0 | 1 | 2 |
|---|---|---|---|
| Pr | $\frac{4}{21}$ | $\frac{7}{21}$ | $\frac{10}{21}$ |