1. _

   (a)

$$\Pr(\bar{A}\cap\bar{B}\cap\bar{C}) = 1-\Pr(A)-\Pr(B)-\Pr(C)+$$
$$+\Pr(A\cap B)+\Pr(A\cap C)+\Pr(B\cap C)-\Pr(A\cap B\cap C)$$
$$1-3\frac{2\cdot 10!}{11!}+3\frac{2^2\cdot 9!}{11!}-\frac{2^3\cdot 8!}{11!}=\frac{5}{9}=55.56\%$$

   (b)

$$\Pr(\overline{A_{12}}\cap\overline{A_{12}}\cap\overline{A_{12}}) = 1-\Pr(A_{12})-\Pr(A_{13})-\Pr(A_{23})+$$
$$+\Pr(A_{12}\cap A_{13})+\Pr(A_{12}\cap A_{23})+\Pr(A_{13}\cap A_{23})+\Pr(A_{12}\cap A_{13}\cap A_{23})$$
$$1-3\frac{2\cdot 10!}{11!}+3\frac{2\cdot 9!}{11!}-0=\frac{28}{55}=50.91\%$$

   Alternate solution (assign each man a RHS neighbor, then permute 8 objects, plus place a man in the rightmost seat and do the same as before with the remaining 2 men):

$$\frac{8\cdot 7\cdot 6\cdot 8!}{11!}+\frac{3\cdot 8\cdot 7\cdot 8!}{11!}=50.91\%$$

2. _

   (a) Divide the deck into 6 parts: ace of spades, ace of diamonds, remaining 2 aces, remaining 12 spades, remaining 12 diamonds, remaining 24 cards (neither aces nor diamonds nor spades); four possibilities (depending on whether you include the ace of spades and/or the ace of diamonds) will then contribute to yield:

$$\frac{\binom{12}{2}\binom{12}{2}\binom{24}{2}+2\cdot 12\binom{12}{2}\binom{24}{3}+2\cdot 12\binom{12}{2}\binom{24}{3}+12^2\binom{24}{4}}{\binom{52}{8}}=1.215\%$$

   (b) We have to consider three cases: 4 singlets, a triplet and a singlet, a quad.

$$\frac{\binom{13}{2}\binom{11}{4}6^2\cdot 4^4+\binom{13}{2}11\cdot 10\cdot 6^2\cdot 4^2+\binom{13}{2}11\cdot 6^2}{\binom{52}{8}}=32.18\%$$

3. _

   (a)

$$\frac{\binom{9}{2,3,4}}{\frac{\binom{12}{4,4,4}}{3!}}=21.82\%$$

Alternate solution: Put Jim into any team; then Joe has 3 out of 11 chances to join him, once done Tom has 8 out of 10 chances to be placed on a different team.

$$\frac{3}{11} \cdot \frac{8}{10} = \frac{12}{55} = 21.82\%$$

(b) This is 3 times the previous answer (the three possibilities are disjoint) plus the probability that they are all on the same team:

$$3 \times \frac{12}{55} + \frac{\frac{\binom{9}{1,4,4}}{2}}{\frac{\binom{12}{4,4,4}}{3!}} = 70.91\%$$

or, using the alternate approach

$$3 \times \frac{12}{55} + \frac{3}{11} \cdot \frac{2}{10} = 70.91\%$$

4.

(a)

$$\left(\frac{Q_{7-2}}{2!} + \frac{Q_{7-1}}{1!} + Q_7\right) \cdot 7! =$$
$$\frac{\left(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)}{2!} 7! + \left(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right) 7! + \left(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!}\right) 7!$$
$$= 4633$$

(b) We have 3 choices to fill the first slot, 2 choices for the last one, and 5! to fill the rest.

$$3 \cdot 2 \cdot 5! = 720$$

5. _

(a)

$$\Pr\left[\bar{A} \cup \bar{B} \cup C\right] = \Pr\left[\overline{A \cap B} \cup C\right] = 1 - \Pr[A \cap B] + \Pr[C] - \Pr\left[\overline{A \cap B} \cap C\right]$$
$$1 - \Pr[A \cap B] + \Pr[C] - \Pr\left[C\right] + \Pr[C \cap A \cap B] = 1 - 0.09 + 0.04 = 95\%$$

or (through complement)

$$\Pr\left[\bar{A} \cup \bar{B} \cup C\right] = 1 - \Pr\left[A \cap B \cap \bar{C}\right] = 1 - \Pr[A \cap B] - \Pr\left[A \cap B \cap C\right] = 95\%$$

(b)

$$\begin{aligned}
\Pr\left[(\bar{A} \cap \bar{B} \cap C) \cup \bar{C}\right] &= \Pr\left[\bar{A} \cap \bar{B} \cap C\right] + 1 - \Pr[C] = \\
\Pr[\overline{A \cup B} \cap C] + 1 - \Pr[C] &= \Pr[C] - \Pr[C \cap (A \cup B)] + 1 - \Pr[C] = \\
1 - \Pr[(A \cap C) \cup (B \cap C)] &= 1 - \Pr[A \cap C] - \Pr[B \cap C] + \Pr[A \cap B \cap C] = 79\%
\end{aligned}$$

or (through complement)

$$\begin{aligned}\Pr\left[(\bar{A}\cap\bar{B}\cap C)\cup\bar{C}\right] &= 1-\Pr\left[(A\cup B\cup\bar{C})\cap C\right] = \\ 1-\Pr\left[(A\cap C)\cup(B\cap C)\cup\Omega\right] &= 1-P\Pr\left[A\cap C\right]-\Pr[B\cap C]+\Pr[A\cap B\cap C] = 79\%\end{aligned}$$

6. _

(a) Buying 29 pieces of fruit and having a choice of apples, bananas, pears and oranges (permuting 29 circles and 3 bars):

$$\binom{32}{3} = 4960$$

(b)

$$\binom{29}{9,5,7,8}\cdot 3^9\cdot 2^5\cdot(-4)^7\cdot 1^8 = -1.031\times 10^{25}$$

7. _Let A mean getting at least one spade and B_i getting i dots.

(a)

$$\begin{aligned}&\Pr(A) = \Pr(B_1)\Pr(A|B_1)+\Pr(B_2)\Pr(A|B_2)+\Pr(B_3)\Pr(A|B_3)\\ &+\Pr(B_4)\Pr(A|B_4)+\Pr(B_5)\Pr(A|B_5)+\Pr(B_6)\Pr(A|B_6) =\\ &\frac{1}{6}\cdot\frac{1}{4}+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{2}}{\binom{52}{2}}\right)+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{3}}{\binom{52}{3}}\right)+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{4}}{\binom{52}{4}}\right)+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{5}}{\binom{52}{5}}\right)+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{6}}{\binom{52}{6}}\right)\\ &+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{5}}{\binom{52}{5}}\right)+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{6}}{\binom{52}{6}}\right) = \frac{220,599}{368,480} = 59.87\%\end{aligned}$$

This can also be done by drawing probability tree (Stage 1 corresponding to number of dots, Stage 2 having 2 branches: one for getting no spades, the other for at least one) and using Bayes' rule.

(b)

$$\begin{aligned}&\frac{\Pr(B_1)\Pr(A|B_1)+\Pr(B_2)\Pr(A|B_2)}{\Pr(A)} =\\ &\frac{\frac{1}{6}\cdot\frac{1}{4}+\frac{1}{6}\cdot\left(1-\frac{\binom{39}{2}}{\binom{52}{2}}\right)}{\frac{220599}{368480}} = 19.24\%\end{aligned}$$