MATH 2F81 SECOND MIDTERM NOVEMBER 12, 2010
Full credit given for 5 correct and complete answers.
One sheet of notes and Maple workspace permitted.
Duration: 2 hours.

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1. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.47$, $\Pr(B) = 0.21$, $\Pr(C) = 0.83$ and $\Pr(D) = 0.55$, find

$$\Pr[(A \cup \bar{B}) \cap \overline{B \cup C} \cap (C \cup \bar{D})]$$

   Hint: find and simplify the complement.

2. Given the following joint probability function

$$f_{xy}(i, j) = c \cdot (2i + j + 1)$$

   of two random variables X and Y, where i and j are two non-negative (note that this *includes* 0) integers such that

$$i + 2j \leq 4$$

   find:

   (a) The value of c,
   (b) ρ_{xy},
   (c) $\mathbb{E}[Y(1 - Y) \mid X = 0]$.

3. Customers arrive randomly, at an average rate of 12.3 per hour. If the store opens at 8:00, find the probability

   (a) of the second customer arriving between 8:06 and 8:12,
   (b) of getting fewer than 5 customers between 8:47 and 9:13.
   (c) If X is the number of customers arriving between 8:47 and 9:13, find expected value and standard deviation of $3 - 2X$.

4. There are 5 blue and 7 red marbles in a box. We randomly draw marbles from this box, one by one and without replacement, until we get the first red marble (at which point we stop). Find the distribution of the number of marbles drawn. Compute the expected value, standard deviation, skewness and kurtosis of this distribution.

5. Consider rolling a die till you get 6 for the fourth time (then stop rolling). Compute

   (a) the probability that this will take between 20 and 30 rolls (inclusive),

   (b) the expected value and standard deviation of the number of rolls,

   (c) the third factorial moment of the corresponding (number-of-rolls) distribution. Hint: This, as we know, is equal to the third derivative of PGF at $z = 1$.

6. Consider playing the following game: 5 cards are dealt randomly from a standard deck, and you receive \$3 for each ace and \$1 for each face card (J, Q, K), but you have to pay \$2 for each spade (this means you get \$1 *net* for the ace of spades and have to pay \$1 net for each of: J, Q and K of spades). Find the expected value and standard deviation of the net win. What is the probability of losing more than \$7?

7. Consider a random variable X with the following probability generating function

$$P(z) = \left(\frac{3+2z}{7-2z}\right)^4$$

   Find

   (a) the mean and standard deviation of X,

   (b) $\Pr(X = 3)$,

   (c) the probability generating function of $2X + 5$.