MATH 2F81 SECOND MIDTERM NOVEMBER 14, 2012
Full credit given for 4 correct and complete answers
One sheet of notes and Maple workspace permitted. **Duration: 90 min.**

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1. There are 2 blue, 5 red, 3 green and 7 yellow marbles in a box. Three marbles are drawn from this box randomly, without replacement. Find the probability

   (a) that they are of three *different* colors,

   (b) of getting more red than blue marbles.

2. Assume that A, B, C and D are four mutually *independent* events, having the probability of 0.32, 0.46, 0.71 and 0.55 respectively. Compute

   (a)
   $$\Pr\left[(A \cap \bar{B}) \cup (B \cap \bar{C}) \cup (C \cap \bar{D}) \cup (D \cap \bar{A})\right]$$

   (b)
   $$\Pr\left[(\bar{A} \cup \bar{B} \cup C) \cap (C \cup \bar{D})\right]$$

   Hint: the complement may be easier.

3. Given the following joint probability function

   $$f_{xy}(i,j) = c \cdot (1 + i^2 + j)$$

   of two random variables X and Y, where i and j are two integers such that

   $$\begin{aligned} -1 &\leq i \leq 2 \\ \max(0,i) &\leq j \leq 2 \end{aligned}$$

   and c is the proper constant (find its value first), compute

   (a) the mean and standard deviation of each (individually) X and Y,

   (b)
   $$\mathrm{Var}(2X - 3Y + 5)$$

4. Consider playing the following game: you pay \$11 to roll three dice and receive one dollar for each dot shown.

   (a) Find the expected value and standard deviation of your *net* win (in one round of this game). Hint: $W = X_1 + X_2 + X_3 - 11$.

   (b) What is the probability of getting all 3 sixes (and winning, net, \$7) in more than two rounds of this game, assuming that 50 rounds are played?

5. Customers arrive at a store randomly, at an average rate of 17.2 per hour. The store opens at 9 am.

   (a) Find the probability that the fifth customer arrives between 9:10 and 9:17 am.

   (b) If X is the number of customers to arrive between 10 am and 10:23 am, find the third factorial moment of X.

6. Two players are randomly dealt 6 cards each (from a standard deck of 52 cards).

   (a) Find the probability that the second player gets the same number of aces as the first player.

   (b) Given that the first player got the king of spades (no clue about the other 5 cards), what is the conditional probability of the second player getting the queen of diamonds? Hint: use $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$.

7. You *are* paid \$1 to play the following game: roll a die 10 times and receive \$8 for each six, but you have to pay \$3 each time the die shows an odd number of dots. Find

   (a) the expected value and standard deviation of your net win,

   (b) the probability of losing money. Hint: use the corresponding PGF and Maple.