1.

(a)

$$\Pr(brg)+\Pr(bry)+\Pr(bgy)+\Pr(rgy) = \frac{2\cdot 5\cdot 3+2\cdot 5\cdot 7+2\cdot 3\cdot 7+5\cdot 3\cdot 7}{\binom{17}{3}} = 36.32\%$$

(b)

$$\Pr(3red)+\Pr(2red)+\Pr(1red\cap 0blue) = \frac{\binom{5}{3}+\binom{5}{2}\cdot 12+5\cdot\binom{10}{2}}{\binom{17}{3}} = 52.21\%$$

2. Pr of A, B, C, D: $0.32, 0.46, 0.71, 0.55$; and of complements: $0.68, 0.54, 0.29, 0.45$ (it helps to get organized).

(a)

$$\begin{aligned} & \Pr(A\cap\bar{B})+\Pr(B\cap\bar{C})+\Pr(C\cap\bar{D})+\Pr(D\cap\bar{A}) \\ & -\Pr(A\cap\bar{B}\cap C\cap\bar{D})-\Pr(B\cap\bar{C}\cap D\cap\bar{A}) \\ = \; & 0.32\times 0.54+0.46\times 0.29+0.71\times 0.45+0.55\times 0.68 \\ & -0.32\times 0.54\times 0.71\times 0.45-0.46\times 0.29\times 0.55\times 0.68 = 89.46\% \end{aligned}$$

as all the remaining intersections are empty.

(b)

$$\begin{aligned} & 1-\Pr\left[(A\cap B\cap\bar{C})\cup(\bar{C}\cap D)\right] = 1-\Pr(A\cap B\cap\bar{C})-\Pr(\bar{C}\cap D)+\Pr(A\cap B\cap\bar{C}\cap D) \\ & = 1-0.32\times 0.46\times 0.29-0.29\times 0.55+0.32\times 0.46\times 0.29\times 0.55 = 82.13\% \end{aligned}$$

3.

	-1	0	1	2	
0	$2c$	c	0	0	$\frac{3}{29}$
1	$3c$	$2c$	$3c$	0	$\frac{8}{29}$
2	$4c$	$3c$	$4c$	$7c$	$\frac{18}{29}$
	$\frac{9}{29}$	$\frac{6}{29}$	$\frac{7}{29}$	$\frac{7}{29}$	

where $c = \frac{1}{29}$.

(a)

$$\begin{aligned}
\mu_x &= \frac{-9+7+14}{29} = \frac{12}{29} = 0.4138 \\
\mu_y &= \frac{8+36}{29} = \frac{44}{29} = 1.517 \\
Var(X) &= \frac{9+7+28}{29} - \left(\frac{12}{29}\right)^2 = \frac{1132}{841} \\
\sigma_x &= \sqrt{\frac{1132}{841}} = 1.160 \\
Var(Y) &= \frac{8+4\times 18}{29} - \left(\frac{44}{29}\right)^2 = \frac{384}{841} \\
\sigma_y &= \sqrt{\frac{384}{841}} = 0.6757
\end{aligned}$$

(b)

$$\begin{aligned}
\text{Cov}(X,Y) &= \frac{-3-8+3+8+28}{29} - \left(\frac{12}{29}\right)\cdot\left(\frac{44}{29}\right) = \frac{284}{841} = 0.3377 \\
\text{Var}(2X-3Y+5) &= 4\cdot\frac{1132}{841} + 9\cdot\frac{384}{841} - 12\cdot\frac{284}{841} = \frac{4576}{841} = 5.441
\end{aligned}$$

4.

(a)

$$\begin{aligned}
\mu_w &= 3\times\frac{7}{2} - 11 = -0.5 \text{ dollars} \\
\sqrt{3\times\frac{35}{12}} &= 2.958 \text{ dollars}
\end{aligned}$$

(b)

$$1-\left(\frac{215}{216}\right)^{50} - 50\cdot\left(\frac{215}{216}\right)^{49}\cdot\frac{1}{216} - \binom{50}{2}\cdot\left(\frac{215}{216}\right)^{48}\cdot\left(\frac{1}{216}\right)^{2} = 0.1653\%$$

5.

(a)

$$\begin{aligned}
&\Lambda_1 = \frac{10}{60}\times 17.2 = \frac{43}{15} = 2.867 \\
&\Lambda_2 = \frac{17}{60}\times 17.2 = \frac{731}{150} = 4.873 \\
&e^{-\frac{43}{15}}\sum_{i=0}^{4}\left(\frac{43}{15}\right)^i/i! - e^{-\frac{731}{150}}\sum_{i=0}^{4}\left(\frac{731}{150}\right)^i/i! = 37.415
\end{aligned}$$

(b)

$$\Lambda = \frac{23}{60} \times 17.2 = \frac{989}{150} = 6.593$$
$$\left(e^{\frac{989}{150}(z-1)}\right)'''\Bigg|_{z=1} = \left(\frac{989}{150}\right)^3 = 286.6$$

6.

(a)

$$\frac{\binom{4}{0}\cdot\binom{48}{6}}{\binom{52}{6}}\cdot\frac{\binom{4}{0}\cdot\binom{42}{6}}{\binom{46}{6}}+\frac{\binom{4}{1}\cdot\binom{48}{5}}{\binom{52}{6}}\cdot\frac{\binom{3}{1}\cdot\binom{43}{5}}{\binom{46}{6}}+\frac{\binom{4}{2}\cdot\binom{48}{4}}{\binom{52}{6}}\cdot\frac{\binom{2}{2}\cdot\binom{44}{4}}{\binom{46}{6}} = 44.21\%$$

(b)

$$\frac{\frac{\binom{50}{5}}{\binom{52}{6}}\cdot\frac{\binom{45}{5}}{\binom{46}{6}}}{\frac{\binom{51}{5}}{\binom{52}{6}}} = \frac{2}{17} = 11.76\%$$

or, more directly (give the first player the king of spades, then deal 6 cards to the second player - the first player can get his remaining 5 cards last):

$$\frac{\binom{50}{5}}{\binom{51}{6}} = \frac{2}{17}$$

7.

$$W = 8X - 3Y + 1$$

(a)

$$\mu_w = 8\cdot 10\cdot\frac{1}{6} - 3\cdot 10\cdot\frac{1}{2} + 1 = -\frac{2}{3}\text{ dollars } = -66.67\text{ cents}$$
$$\sigma_x = \sqrt{8^2\cdot 10\cdot\frac{1}{6}\cdot\frac{5}{6} + (-3)^2\cdot 10\cdot\frac{1}{2}\cdot\frac{1}{2} + 2\cdot 8\cdot(-3)\cdot(-10\cdot\frac{1}{6}\cdot\frac{1}{2})} = 12.30\text{ dollars}$$

(b)

$$P_w(z) = z\cdot\left(\frac{z^{-3}}{2} + \frac{1}{3} + \frac{z^8}{6}\right)^{10}$$

When expanded, this yields $\Pr(W < 0) = 54.41\%$.