MATH 2F81 FIRST MIDTERM OCTOBER 10, 2006
Full credit given for 6 correct and complete answers (out of 8)
Open-book exam. Duration: 2 hours

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1. Three friends and 17 other boys are to be randomly divided into 5 teams of 4 players each. What is the probability that the three friends find themselves separated, each playing for a different team?

2. If 7 dice are rolled, what is the probability of getting *exactly* one triplet (three identical numbers)?

3. When
$$(1 - 3x + 4y^2 - 2z^3)^{157}$$
is fully expanded, what is the coefficient of $x^{15}y^{24}z^{33}$? Also, how many terms will be in such expansion?

4. Express
$$(A \cup C) \cap (B \cup \overline{C \cup D})$$
as a union of 'simple' intersections (intersections consisting of any number of letters - including one, i.e. *without* $\cap$ - some letters may have a bar).

5. If
$$\begin{aligned} P(A) &= 0.31, \quad P(B) = 0.33, \quad P(C) = 0.37 \\ P(A \cap B) &= 0.11, \quad P(A \cap C) = 0.13, \quad P(B \cap C) = 0.12 \\ &\quad \text{and } P(A \cap B \cap C) = 0.05 \end{aligned}$$
compute
$$P\left[(A \cap \overline{B}) \cup (B \cap \overline{A \cup C})\right]$$

6. 4 friends and 9 other boys ('strangers') are to be randomly seated at a round table. Find the probability that each friend will sit next to *exactly* one stranger (and one friend).

7. Five cards are dealt randomly from an ordinary deck of 52 cards. What is the probability of getting exactly one spade and 2 diamonds?

8. Four letters (each written to a different person) are *randomly* placed into four envelopes (these already have the persons' addresses on it). What is the probability that

   (a) no letter is placed correctly

   (b) one letter is placed correctly

   (c) two letters are placed correctly

   (d) three letters are placed correctly

   (e) all four letters are placed correctly?