1 Counting formulas (number of ways to):

1.1 Select r out of n symbols: n^r, C_r^n, P_r^n, C_{n-1}^{r+n-1}

Binomial expansion: $(x+y)^n = ...$

1.2 Permute $aaabbc$, or partition a group of N into k subgroups: $\binom{N}{n_1, n_2, ..., n_k}$

Multinomial expansion: $(x+y+z)^n$

Typical questions:

1. Expand $(1 - 2x + 3x^2 + ..)^{17}$
2. Probability of 2 quadruplets, 1 triplet, etc. when rolling a die 10 times.

2 Random variables

2.1 Sample space (simple events and their probabilities)

Be able to list explicitly in simple cases (flipping a coin 3 times, rolling two dice, etc.)

2.2 Events and Boolean expressions - basic probability rules - mutual independence

Compute $\Pr\{(A \cap \overline{B}) \cup ...\}$ in both the dependent and independent case.

Be able to introduce your own A, B, C when necessary (spouses to sit next to each other, etc).

3 Conditional probabilities

3.1 Probability tree (product rule)

This time, often combined with binomial (multinomial) or hypergeometric distributions!

E.g. select a box, draw 5 marbles from it, how many are red?

3.2 Total probability formula

Probability of 2 red marbles (regardless which box).

3.3 Bayes rule

Given 2 were red, probability that Box I was selected.

4 Random variable(s) - integer type

4.1 Distribution

Given by table or formula (probability function) - convert to table.

Based on bivariate table, find marginal distributions, any **conditional** distribution (of X given $Y = 1$).

Construct a distribution of $U = (X - 2)^2$.

4.2 Expected value(s)

Univariate case: Mean, variance (standard deviation), skewness, $\mathbb{E}(\frac{1}{1+X})$.

Bivariate case: Also covariance, correlation coefficient, $\mathbb{E}(\frac{Y}{X+Y})$.

Mean and variance of a **linear combination** of random variables (formulas).

Conditional expected value (use the corresponding conditional distribution).

4.3 Moment generating function

Extract simple moments (by differentiating).
The $M_{aX+b}(t)$ formula.
MGF of an independent sum (product of individual MGFs).

5 Special distributions - integer type

5.1 Binomial (Bernoulli) - # of successes in n trials

Be able to recognize the experiment, identify n and p.
Compute individual probabilities, mean and variance (formulas).
Extension to multinomial - individual probabilities, **covariance** formula.

5.2 Negative binomial (Geometric) - # of trials to get k^{th} success

Recognize experiment, identify k and p.
Compute individual probabilities, **cumulative** probabilities, mean and variance (formulas).

5.3 Poisson - # of arrivals

Find the corresponding λ (rate times interval, or length, or area).
Individual probabilities, mean, variance.
Probability of 4^{th} arrival between 10:30 and 11:00.

5.4 Hypergeometric - # of red marbles

Sampling without replacement, parameters: n, K and N.
Individual probabilities, mean and variance.
Extension to multivariate case (blue, green and yellow marbles) - individual probabilities, **covariance**.

6 Random variables - continuous type

Specified by probability density function instead of individual probabilities (always by formula - sometimes **piecewise**)!

Construct distribution function $F(x)$ - also piecewise, if necessary, find **median** and quartiles.

Everything else is an exact analog of integer case - summation changes to integration.

Bivariate distribution - understand the concept of **conditional** and **marginal range**.

7 Special distributions - continuous type

7.1 Uniform

Probability of being in some interval; mean and variance formulas.

7.2 Normal

Standard Z - find probability of any interval, from tables (linear interpolation).

General - find probability of any interval by converting to Z.

Used as **approximation** to many other distributions, also to any large independent **sum** of random variables - **continuity correction** when appropriate!

7.3 Exponential

Formulas: $f(x)$, $F(x)$, mean, standard deviation, median, moment generating function.

Be able to compute any related probability.

The smallest value in a random independent sample of size n from this distribution is also exponentially distributed, but its mean is n times smaller.

7.4 Gamma

defined as a sum of k independent random variables of the exponential type (time till the arrival of the k^{th} customer).

Remember the mean and standard deviation formulas, also be able to find the probability of any interval - based on $F(x)$!

7.5 Bivariate Normal

has five parameters. Marginals are univariate Normal.

Conditional distributions are also Normal. Be able to find the conditional mean and standard deviation - based on this, any conditional probability.