MATH 2F81SECOND MIDTERMNOVEMBER 11, 2011Full credit given for 5 correct and complete answers.One sheet of notes and Maple workspace permitted.Duration: 2 hours.

1. If A, B, C and D are mutually *independent*, and Pr(A) = 0.46, Pr(B) = 0.23, Pr(C) = 0.84 and Pr(D) = 0.51, find

(a)

$$\Pr[(A \cap \bar{B}) \cup (B \cap C) \cup (C \cap \bar{D})]$$

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(b) and (don't fall into a trap)

$$\Pr[(\bar{A} \cup B) \cap \overline{B \cup C} \cap (\bar{C} \cup D)]$$

2. Given the following joint probability function

$$f_{xy}(i,j) = c \cdot (2i^2 + j + 1)$$

of two random variables X and Y, where i and j are two integers such that

$$\begin{array}{rrr} -1 & \leq & i \leq 2 \\ \max(1,i) & \leq & j \leq 2 + \mid i \end{array}$$

and c is the appropriate constant, find:

- (a)  $\operatorname{Cov}(X, Y)$ ,
- (b)  $\mathbb{E}[X^2 4 \mid Y = 3].$
- 3. Rolling a die 4 times, what is the probability that
  - (a) a six will be *immediately* followed by a one (at least once, during these 4 rolls),
  - (b) a six will be followed (not necessarily immediately) by a one (at least once).

4. Given that X has the following distribution

X =	-2	0	1	3	5	8
Pr:	0.15	0.25	0.30	0.15	0.10	0.05

compute

- (a) its mean, standard deviation and skewness,
- (b) its third *factorial* moment.
- 5. Three cards are randomly dealt from a standard deck of 52 cards, followed by rolling a die as many times as the number of spades obtained.
  - (a) Compute the probability of getting more than 3 dots in total.
  - (b) Given that we got more than 3 dots in total, find the conditional probability of having dealt more than one spade.
- 6. A gambler is charged \$4 to play the following game: 5 cards are dealt randomly from a standard deck, and he receives \$3 for each face card (J, Q, K). Find
  - (a) the expected value and standard deviation of his net win (in one such game),
  - (b) the probability that he will be losing money after 5 *independent* rounds of this game.
- 7. Consider two *independent* random variables  $X_1$  and  $X_2$ , each having the following probability generating function

$$P(z) = \exp\left(5 \times \frac{z-1}{2-z}\right)$$

Find

- (a) the mean and standard deviation of  $X_1 2X_2$ ,
- (b)  $\Pr(5 < X_1 + X_2 < 15).$