1. **a.** Using A, B and C for the first, second and third couple (respectively) sitting next to each other, we get

$$\Pr(A \cup B \cup C) = 3\Pr(A) - 3\Pr(A \cap B) + \Pr(A \cap B \cap C)$$

$$= 3\frac{2 \cdot 14!}{15!} - 3\frac{2^2 \cdot 13!}{15!} + \frac{2^3 \cdot 12!}{15!} = 34.58\%$$

b.

$$\frac{8! \cdot 7!}{15!} = 0.01554\%$$

2.

a.

$$\frac{\binom{12}{3}5^9}{6^{12}} = 19.74\%$$

b.

$$\frac{\binom{12}{3}\binom{9}{2}4^7}{6^{12}} = 5.961\%$$

c.

$$\frac{\binom{6}{2}\binom{12}{4,4,4}(4^4 - 4)}{6^{12}} = 6.017\%$$

3.

a.

$$(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} - \frac{1}{9!} + \frac{1}{10!} - \frac{1}{11!})11!$$
$$= 14684570 \simeq 1.468 \times 10^7$$

Note that $e^{-1} \cdot 11!$ would have been an excellent approximation.

b.

$$(\frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!}) \cdot 11!/4!$$
$$= 611820 \simeq 6.118 \times 10^5$$

The $e^{-1} \cdot 11!/4!$ approximation would yield 6.119×10^5.

4.

$$\Pr[(A\cup\bar{B}\cup C)\cap(A\cup\bar{B}\cup\bar{C})]$$
$$=1-\Pr[(\bar{A}\cap B\cap\bar{C})\cup(\bar{A}\cap B\cap C)]$$
$$=1-\Pr(\bar{A}\cap B\cap\bar{C})-\Pr(\bar{A}\cap B\cap C)$$
$$=1-\Pr(\bar{A}\cap B)+\Pr(\bar{A}\cap B\cap C)-\Pr(B\cap C)+\Pr(A\cap B\cap C)$$
$$=1-\Pr(B)+\Pr(A\cap B)+\Pr(B\cap C)-\Pr(A\cap B\cap C)-\Pr(B\cap C)+\Pr(A\cap B\cap C)$$
$$=1-\Pr(B)+\Pr(A\cap B)=1-0.33+0.11=0.78$$

Alternate solution:

$$\Pr[(A\cup\bar{B}\cup C)\cap(A\cup\bar{B}\cup\bar{C})]$$
$$=\Pr[(A\cup\bar{B})\cup(C\cap\bar{C})]$$
$$=\Pr[(A\cup\bar{B})\cup\varnothing]=\Pr(A\cup\bar{B})$$
$$=\Pr(A)+\Pr(\bar{B})-\Pr(A\cap\bar{B})$$
$$=\Pr(A)+1-\Pr(B)-\Pr(A)+\Pr(A\cap B)$$
$$=1-\Pr(B)+\Pr(A\cap B)=1-0.33+0.11=0.78$$

(using Venn diagram is also OK).

5.

a.

$$\frac{3\cdot 12\cdot\binom{36}{3}+\binom{36}{4}}{\binom{52}{5}}=12.16\%$$

b.

$$\frac{3\cdot 12\cdot\binom{36}{3}}{\binom{52}{5}}\cdot\frac{2\cdot 11\cdot\binom{33}{3}+\binom{33}{4}}{\binom{47}{5}}+\frac{\binom{36}{4}}{\binom{52}{5}}\cdot\frac{3\cdot 12\cdot\binom{32}{3}}{\binom{47}{5}}=1.302\%$$

6. Same number of aces:

$$\frac{\binom{48}{5}}{\binom{52}{5}}\cdot\frac{\binom{43}{5}}{\binom{47}{5}}+\frac{4\binom{48}{4}}{\binom{52}{5}}\cdot\frac{3\binom{44}{4}}{\binom{47}{5}}+\frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}}\cdot\frac{\binom{45}{3}}{\binom{47}{5}}=\frac{26711}{54145}$$

Answer:

$$\frac{1-\frac{26711}{54145}}{2}=25.33\%$$

7. This can be made into a probability tree

0, >1	$\frac{\binom{48}{5}}{\binom{52}{5}} \cdot 0$	○	
0, E	$\frac{\binom{48}{5}}{\binom{52}{5}} \cdot 1$		
1, >1	$\frac{4\binom{48}{4}}{\binom{52}{5}} \cdot 0$	○	
1, E	$\frac{4\binom{48}{4}}{\binom{52}{5}} \cdot 1$		
2, >1	$\frac{6\binom{48}{3}}{\binom{52}{5}} \cdot \frac{1}{4}$	○	
2, E	$\frac{6\binom{48}{3}}{\binom{52}{5}} \cdot \frac{3}{4}$		
3, >1	$\frac{4\binom{48}{2}}{\binom{52}{5}} \cdot \frac{1}{2}$	○	√
3, E	$\frac{4\binom{48}{2}}{\binom{52}{5}} \cdot \frac{1}{2}$		
4, >1	$\frac{48}{\binom{52}{5}} \cdot \frac{11}{16}$	○	√
4, E	$\frac{48}{\binom{52}{5}} \cdot \frac{5}{16}$		

a.

$$\frac{6\binom{48}{3}}{\binom{52}{5}} \cdot \frac{1}{4} + \frac{4\binom{48}{2}}{\binom{52}{5}} \cdot \frac{1}{2} + \frac{48}{\binom{52}{5}} \cdot \frac{11}{16} = \frac{9411}{866\,320} = 1.086\%$$

b.

$$\frac{\frac{4\binom{48}{2}}{\binom{52}{5}} \cdot \frac{1}{2} + \frac{48}{\binom{52}{5}} \cdot \frac{11}{16}}{\frac{9411}{866\,320}} = 8.108\%$$