1. Calculate the probability that at least 31 rolls of a die will be needed to get 5 sixes.

2. One has to pay \$4 to play the following game: A die is rolled and the player receives \$12 for a six, \$6 for a five, \$3 for a four, and \$1.50 for a three (nothing for one or two). Find the mean and standard deviation of the net win in one round of this game.

3. Customer arrive at a gas station at a rate of 17 cars per hour. Using the Poisson distribution, compute:

   (a) the expected number of cars served during an 8 hour period, and the corresponding standard deviation,

   (b) the probability of getting more than 3 cars during the next five minutes.

4. Consider the following game: A player pays \$5 and is then allowed to draw five marbles from a box containing one gold, 4 silver, 10 bronze and 25 red marbles. He receives \$10 for a gold marble, \$3 for a silver marble and \$1 for a bronze marble. Compute:

   (a) the expected win (loss) in a single game and the corresponding standard deviation,

   (b) the probability of winning (net) at least \$15 in a single game.

5. A bivariate distribution is defined by the following joint probability function:

   $$f_{XY}(i,j) = c(1+i+j^2) \quad \text{where} \quad 0 \le j \le |\,j\,| \quad \text{with} \quad -1 \le j \le 2$$

   and c is the appropriate constant. Find:

   (a) $\Pr(X+Y \le 1)$,

   (b) $\mathrm{Var}(Y)$,

   (c) $\mathrm{Cov}(X,Y)$,

   (d) $\mathbb{E}(\frac{1}{1+X}\,|\,Y=2)$.

6. Consider rolling 10 regular dice. What is the probability of getting:

(a) two quadruplets of identical numbers,

(b) at least 3 sixes,

(c) exactly 3 sixes and 3 fives.

7. A random variable has the following moment generating function:

$$M(t) = \frac{e^{-2t}}{(1-5t)^3}$$

Find:

(a) the corresponding mean and standard deviation,

(b) the moment generating function of $3X + 2$.

8. Consider flipping 3 coins and a 'trial' and getting three heads a 'success'. The 3 coins are then flipped, repeatedly, until 5 successes are generated. Find:

(a) The expected number of trials and the corresponding standard deviation.

(b) The probability that this will take fewer than 30 trials.

9. A tetrahedron (a die with four sides marked 1, 2, 3 and 4) is rolled once. Depending on the number obtained, a coin is tossed that many times.

(a) Construct the bivariate distribution of X (the number generated by rolling the die) and Y (the total number of heads obtained in the second part of the experiment).

(b) Find the variance of Y.

10. Five dice are rolled, repeatedly, till the all show a different face. This is then repeated three more times (for the total of four). Find:

(a) The expected number of trials (5 die rolls), and the corresponding standard deviation.

(b) The probability that we need to roll, altogether, more than fifty times.

11. On a certain road potholes occur, randomly, at the average rate of 12.6 per 10 km. Consider a 5 km stretch of this road. Using Poisson distribution, find:

    (a) The expected number of potholes, and the corresponding standard deviation.

    (b) The probability of finding fewer than 4 potholes.

    (c) Given that there are fewer than 4 potholes, what is the conditional probability of finding exactly 2?

12. Consider two (rather unusual) dice, each with 1 dot on three of its sides, 2 dots on one side and 3 dots on the remaining two sides. The two dice are rolled. Let X be the (absolute) difference of the number of dots shown, and Y be the smaller of the two numbers. Construct a table of the joint distribution of X and Y. Based on this, compute $Cov(X, Y)$.

13. If 11 dice are rolled, what is the probability of getting:

    (a) exactly 3 sixes,

    (b) at least 2 sixes,

    (c) exactly 1 six and 7 small values ('small' means less the four).

    (d) 2 pair, 1 triplet and 1 quadruplet of identical values.

14. There are 3 red, 7 blue and 5 green marbles in a box. Four are selected at random (without replacement). If a player gets paid \$2 for each blue marble and \$5 for each red marble, but has to pay \$6 for each green marble selected, calculate the expected value and the standard deviation of his net win. What is the probability of winning (net) more than \$15?

15. Consider paying \$5 to play the following game: A die is rolled five times and you are paid \$3 for each six, \$1 for each five and four (nothing for the other 3 numbers). Calculate:

    (a) Expected loss.

    (b) The corresponding standard deviation.

(c) Probability of losing exactly \$2.

16. The number of fatal accidents in a province is a random variable with the mean value of 3.7 per week. Find the probability of having

    (a) fewer than 5 fatal accidents in a week,
    (b) fewer than 5 fatal accidents in two weeks,
    (c) more than two accidents in 3 days.

17. A team will win a game with the probability of 0.32, lose with the probability of 0.41 and tie with the probability of 0.27. They play a series of 6 games.

    (a) If the team is awarded three points for each win and one point for each tie, what is the expected value and standard deviation of the number of points it will earn?
    (b) Calculate the probability that the series will end in a draw (same number of wins and losses).

18. Given the following bivariate probability function

$$f_{XY}(i,j) = c\,\frac{i}{i+j} \qquad \text{where } 1 \le i \le 3 \quad \text{and} \quad i-1 \le j \le i+1$$

find:

    (a) The value of c,
    (b) $\mathrm{Var}(Y)$,
    (c) $\mathbb{E}(X^3|Y=1)$.

19. Pay \$35 to play the following game: A die is rolled three times and you get paid \$2 for each dot shown in the first roll, \$3 for each dot shown in the second roll, and \$5 for each dot shown in the third roll. What is the expected net win (loss) of this game, and the corresponding standard deviation?

20. An office gets on the average 3.7 phone calls per hour. Compute the probability that, during the next half hour, there will be:

(a) exactly 3 phone calls,

(b) at least 3 phone calls.

(c) If the office opens at 8:00, what is the probability of the third phone call of the day arriving between 8:30 and 8:45?

21. Two coins are flipped (this is called a trial), repeatedly, until double heads appear for the fourth time.

    (a) What is the expected number of trials, and the corresponding standard deviation.

    (b) What is the probability that this takes exactly 10 trials?

    (c) Anywhere between 30 and 50 trials (inclusive)?

22. Six marbles are randomly drawn (without replacement) from a box containing 6 red, 9 blue, 5 green and 7 yellow marbles. Compute:

    (a) The probability of getting at least 2 yellow marbles.

    (b) The probability of getting exactly 2 yellow and 2 blue marbles.

    (c) $\mathrm{Var}(X+Y)$, where X is the number of green, and Y the number of blue marbles in the sample.

    (d) $\mathrm{Var}(X-Y)$.

23. A random variable X has e^{2t^2-3t} as its moment generating function. Find the corresponding mean, standard deviation and skewness. What is the moment generating function of $Y = \frac{X-1}{4}$?

24. Consider playing the following game: Pay \$30 to roll a die until the 3^{rd} six appears, being paid \$2 for each roll. What is the expected value and standard deviation of your net win? What is the probability of winning

    (a) exactly \$20,

    (b) more than \$20.

25. When 8 cards are dealt, randomly, from a standard deck of 52 cards, what is

(a) the variance of the total number of spaced and aces (the ace of spades counts both as a spade and an ace, of course),

(b) the correlation coefficient between the number of spades and the number of diamonds,

(c) the probability of getting exactly 2 spades and 3 diamonds,

(d) the probability of getting exactly 4 spades and 2 aces.

26. Consider flipping a coin four times. Construct the bivariate distribution of X (the length of the longest sequence of consecutive heads; 0 if no heads observed) and Y (the length of the longest sequence of consecutive tails). Also, compute $\mathrm{Cov}(X, Y)$.

27. Suppose five cards are dealt from an ordinary deck of 52 cards, and you are paid \$3 for each spade, but you must pay \$3 for each diamond and club. What is the expected a amount of money you lose, and the corresponding standard deviation. What is the probability of breaking even?

28. Given the following joint probability function

$$f_{XY}(i,j) = c(i+j) \qquad \text{where } i \text{ and } j \text{ are integers such that} \quad i^2+j^2 \le 9$$

find:

(a) the value of c,

(b) the marginal distribution of X,

(c) $\mathbb{E}(Y \mid x = 1)$,

(d) $\Pr(X + Y \le 2)$.