MATH 2P82 Assignment 1

Answer all ▶ labelled questions.
Those marked by ▷ are optional, intended for extra practice only.
Using Maple, plot the pdf of each answer.

1. ▷ Let X have the gamma(2,1) distribution. Find both the distribution function and the probability density function of $Y = \dfrac{X}{1+X}$.

2. ▶ If X is a random variable with the following probability density function:
$$f(x) = c \cdot x \qquad 0 < x < 1$$
where c is an appropriate constant, what is the distribution function and the corresponding pdf of $Y = \dfrac{X}{1+X}$?

3. ▷ The distribution of X has the following probability density function:
$$f(x) = \frac{1}{2\sqrt{x}} \qquad 0 < x < 1$$
Find the probability density function of $Y = \dfrac{1}{\sqrt{X}}$.

4. ▶ X has the Cauchy distribution with the median equal to 0 and the quartile deviation of 1.
Find and identify the distribution of $Y = \dfrac{1}{X}$.

5. ▶ If X has the exponential distribution with $\beta = 1$, find the probability density function of $Y = \dfrac{X+1}{X}$. What is the median of the new distribution? Hint: Find the median of X first.

6. ▷ X has the exponential distribution with $\beta = 2$.
Find the distribution of $Y = e^{-X}$.

7. ▷ X has the chi-squared distribution with 3 degrees of freedom.
Find the distribution of $Y = \dfrac{1}{X}$.

8. ▶ X_1 and X_2 are two independent random variables, each from a uniform distribution, with the range of (0,1) and (0,2), respectively.

   Find and sketch the probability density function of $X_1 + X_2$.

9. ▶ Two random variables X and Y have a constant joint probability density function in the following region: $\begin{cases} x^2 + y^2 < 1 \\ y > 0 \end{cases}$ (upper half of the unit disk), zero elsewhere.

   Find and identify the distribution of $\dfrac{X}{Y}$.