MATH 2F82   Assignment 3

1. Consider a RIS of size 218 from a distribution with the following pdf:

$$f(x) = \begin{cases} \frac{3}{2}x^2 & 0 < x < 1 \\ \frac{3}{2}(x-2)^2 & 1 < x < 2 \end{cases}$$

   (a) Using the normal approximation, find the probability that the sample median will fall between 0.99 and 1.02. Also, find the quartile deviation of the above distribution.

   (b) Using the normal approximation, find the probability that the sample mean will fall between 0.99 and 1.02.

2. Consider a RIS of size 4 from the gamma(2,1) distribution. Find:

   (a) $\Pr\{X_{(2)} < 1.8\}$
   (b) $\mathbb{E}\{X_{(2)}\}$

3. Consider RIS of size 301 from a distribution having the following pdf:

$$f(x) = \frac{1}{2\sqrt{x}} \quad 0 < x < 1$$

   Compute (using the normal approximation) the probability that the sample median will have a value smaller than 0.29. Also, find the quartile deviation of the above distribution.

4. Using the distribution of the previous question, but reducing the sample size to 8, find:

   (a) $\Pr\{X_{(5)} > 0.6\}$
   (b) $\mathbb{E}\{X_{(5)}\}$
   (c) $\mathrm{Var}\{X_{(5)}\}$

5. For a random independent sample of size n from a distribution, the corresponding **sample range,** say R, is defined as $X_{(n)} - X_{(1)}$ (the distance from the smallest to the largest observation). Assuming that we are sampling a uniform distribution with the usual 0 to 1 limits, find the probability density function of R. Identify the resulting distribution. Also, when $n = 7$, find $\mathrm{Cov}(X_{(1)}, X_{(7)})$ and $\Pr(X_{(1)} < 0.1 \cap X_{(7)} > 0.9)$.