MATH 2F82 Assignment 4

1. Consider a sample of size n from a distribution with the following pdf:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{when} \quad a < x < b \\ 0 & \text{otherwise} \end{cases}$$

As an estimator of the parameter a, is the first order statistic $X_{(1)}$

(a) unbiased?

(b) asymptotically unbiased?

(c) consistent?

Substantiate each of your answers.

2. Find the Rao-Cramer bound for the variance of any unbiased estimator of θ (a positive number), of the following distribution:

$$f(x) = \frac{3x^2}{\theta} \exp(-\frac{x^3}{\theta}) \qquad \text{where} \qquad x > 0$$

3. Continuing the previous question: Find a sufficient statistics for estimating θ.

4. Still continuing: Convert the sufficient statistic of the previous question into and unbiased estimator of θ, and find this estimator's efficiency.

5. Consider the usual exponential distribution defined by: $f(x) = \frac{1}{\beta} \exp(-\frac{x}{\beta})$ for $x > 0$. One can easily verify that its median $\tilde{\mu}$ equals $\beta \cdot \ln(2)$.

(a) For a sample of size 11, is the sample median an unbiased estimator of $\tilde{\mu}$? If not, how would you make it into an unbiased estimator? Note: To help Maple with the corresponding integration, you may have to change the integration variable from x to $u = \exp(-\frac{x}{\beta})$.

(b) Optional: Repeat for a sample of size $2k+1$, where k is a positive integer.