1. Using Gaussian elimination with full pivoting (the computation should be carried out with decimals, not fractions), solve the following set of ordinary equations:

$$\begin{bmatrix} -9. & 41. & -5. & -149. \\ -80. & -8. & 384. & 4. \\ 8. & -9. & 921. & -49. \\ 41. & -135. & -3. & -54. \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 82. \\ -2. \\ 581. \\ 147. \end{bmatrix}$$

   Also: compute the four errors (difference between the left and right hand side of each equation) as a measure of the achieved accuracy.

2. Using the Gram-Schmidt procedure, find the first four (up to and including cubic) orthogonal polynomials over the $[0,1]$ interval, using $\omega(x) = \frac{1}{\sqrt{x}}$ as the weight function. This means that the polynomials must meet

$$\int_0^1 \frac{\phi_i(x)\phi_j(x)}{\sqrt{x}} dx = 0$$

   wheever $i \neq j$. Compute the corresponding α's, by

$$\alpha_i = \int_0^1 \frac{\phi_i(x)^2}{\sqrt{x}} dx$$

3. Use the trapezoidal rule with $n = 1$, 2, 4 and 8 subintervals to evaluate, as accurately as possible (i.e. performing all stages of Romberg's technique), the following integral

$$\int_2^5 \frac{1}{\sqrt{1+x^2}}\, dx$$

   Compare your answer with the 'exact' one, computed by Maple.