1. Find the L-U decomposition of

$$\mathbb{A} \equiv \begin{bmatrix} 2 & -1 & 0 & 0 \\ 3 & -2 & 5 & 0 \\ 0 & -3 & 4 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Use the result to find the four-component vector $\mathbf{x}$ which solves

$$\mathbb{A}\,\mathbf{x} = \begin{bmatrix} 8 \\ 13 \\ 4 \\ -5 \end{bmatrix}$$

Give all details of your computation (ie. you may have to do this 'by hand'); also, verify (by Maple) that the answers ($\mathbb{L}$, $\mathbb{U}$ and $\mathbf{x}$) are correct.

2. Using $n = 4$ and $n = 8$ for the number of subintervals, find an approximate solution to

$$y'' - \frac{y'}{1+x^2} + \frac{y}{\sqrt{3+x}} = \frac{x}{2}$$

with $y(0) = 2$ and $y(2) = 3$. Improve the values of $y(0.5)$, $y(1)$ and $y(1.5)$ by Richardson extrapolation. Point plot the $n = 8$ solution (including the end points).