MATH 3P60 SECOND MIDTERM NOVEMBER 24, 2005
Full credit given for three correct and complete answers.
If possible, use Maple 8. Save your work in *.mws format.
Open-book exam. Duration: 80 minutes

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1. Design a 2-point *Gaussian* formula (use decimals, not exact numbers) to approximate
$$\int_1^2 \frac{y(x)}{1+x}dx$$
where $y(x)$ is an arbitrary function and $\frac{1}{1+x}$ is the weight function.
Apply your formula to
$$\int_1^2 \frac{e^{-x}}{1+x}dx$$

2. Solve, numerically
$$y'' + \sin(x)y' - \frac{y}{1+x^3} = e^{-x}$$
where $y(-1) = 0.5$ and $y(3) = 4$, by subdividing the $(-1,3)$ interval first into 4, and then into 12 subintervals. Improve the $y(0)$, $y(1)$ and $y(2)$ answers by Richardson extrapolation.

3. Fit
$$\tan x$$
in the $(0, \frac{\pi}{4})$ interval by the best **cubic** polynomial, utilizing Chebyshev polynomials. Find the largest (absolute) error of your fit.

4. Find, to an 8 digit accuracy, at least two distinct solutions to
$$\begin{aligned} \frac{x^2}{y+\sin(x+y)} &= \ln(x^2+y^4) \\ (y-x^2)e^{-x} &= 3x-4y^2 \end{aligned}$$

5. Evaluate
$$\int_{-1}^2 \frac{e^{-x}}{2+\cos x}dx$$
using the Simpson composite rule with $n=2$, $n=6$ and $n=18$. Use the three answers to improve the result by performing two stages of Romberg's algorithm.