

1. For a bivariate distribution with the following joint pdf

$$f(x, y) = 12xy \quad \begin{cases} 0 < x < 1 \\ 0 < y < 1 - x^2 \end{cases}$$

(zero otherwise) find

- (a) the marginal pdf of Y (don't forget to spell out its support)
 - (b) $\mathbb{E}(X \mid Y = \frac{1}{2})$
 - (c) $\mathbb{E}(X + Y)$
 - (d) $\Pr(X > 2Y)$
2. Repeat with

$$f(x, y) = \exp(-x) \quad x > y > 0$$

3. A random variable X has the following pdf:

$$f = \begin{cases} 3c \cdot (1 + x) & \text{when } -1 < x < 0 \\ 2c & \text{when } 0 < x < 1 \\ c \cdot (2 - x)^2 & \text{when } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (a) the value of c ,
 - (b) the distribution function $F(x)$,
 - (c) $\Pr(\frac{1}{2} < X < \frac{3}{2})$,
 - (d) the mean and standard deviation of X .
4. Consider a RIS of size 100 from the distribution of the previous question. Using the Normal approximation (a.k.a. Central Limit Theorem) find the probability that the corresponding *sample mean* \bar{X} will have a value bigger than 0.35.
5. Let X and Y be independent, Normally distributed random variables with $\mu_x = 3$, $\mu_y = -2$, $\sigma_x = 5$ and $\sigma_y = 5$. Compute:
- (a) $\Pr\{-9 < X < 9 \cap -9 < Y < 9\}$,
 - (b) $\Pr\{(X - 3)^2 + (Y + 2)^2 < 10\}$.