

1. Find

$$\mathbb{E}[\exp(t_1 Z_1 + t_2 Z_2)]$$

where Z_1 and Z_2 are RVs having the standardized bivariate Normal distribution with the correlation coefficient of ρ .

Also: prove that when X has Exponential distribution with parameter β and x and x_0 are both positive, then

$$\Pr(X - x_0 < x \mid X > x_0) = \Pr(X < x)$$

2. Consider a RIS of size 205 from the bivariate distribution of Question 2 of the previous assignment. Using the *univariate* version of CLT, find an approximation to

$$\Pr(0.9 < \bar{Y} < 1.1)$$

3. and also to

$$\Pr(\bar{X} > 2.1\bar{Y})$$

Hint: first find the expected value and variance of $U = X - 2.1Y$.

4. Continuation: Using the *bivariate* version of CLT approximate

$$\Pr(\bar{X} > 1.9 \cap \bar{Y} < 0.9)$$

Hint: now you will need the two expected values, both variances, and $\text{Cov}(X, Y)$.

5. If X and Y are jointly Normal with $\mu_x = 3.1$, $\mu_y = 103$, $\sigma_x = 1.2$, $\sigma_y = 28$, and $\rho_{xy} = -0.89$, calculate

$$\Pr(X < 3 \mid Y = 77)$$

Also, find the *joint* distribution of $U \equiv 25X - Y + 32$ and $V \equiv X + \frac{Y}{28} - 3$ (just spell out its name and parameter values), and the probability of $U < -2.4$.