BROCK UNIVERSITY

Final Examination: April 2017	Number of Pages: 3
Course: MATH 3F85	Number of students: 22
Date of Examination: Apr 25, 2017	Number of Hours: 3
Time of Examination: 19:00-22:00	Instructor: J. Vrbik

Open-book exam. Full credit given for 18 (out of 27) correct (to at least 4 significant digits), complete and **fully simplified** answers. Numerical answers should be given in **decimal** form. Students are allowed to use *basic* features of Maple **only** (no internet searching, etc.). All final answers must be entered in your booklet; Maple work to be either printed and attached, or sent to jvrbik@brocku.ca

1. Consider two random variables X and Y having the following joint pdf

$$f(x,y) = \begin{cases} \dfrac{3(x^2+y)}{2(3\pi+8)} & \text{when } x^2+y^2<4 \quad \text{and} \quad y>0 \\ 0 & \text{otherwise} \end{cases}$$

Find the

(a) conditional pdf of X (and its support), given that $Y=1$,

(b) median and quartile deviation of Y,

(c) $\text{Var}(X^2)$,

(d) $\Pr(X<Y)$,

(e) pdf of $U=\sqrt{X^2+Y^2}$ and its support (hint: find its cdf first).

2. Consider a random variable X with the following pdf

$$f(x) = \begin{cases} \left(\dfrac{1+x}{2}\right)^2 & -1<x\le 0 \\ c+2x(1-x) & 0<x\le 1 \\ \dfrac{2-x}{3} & 1<x\le 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the

(a) value of c,

(b) $\Pr(X^2<0.6)$.

Assuming a RIS (random independent sample) of size 73 from this distribution, find

(c) $\Pr(\tilde{X}<0.6)$, where $\tilde{X}$ stands for the sample median, using the Normal approximation (*not* the exact answer),

(d) $\Pr(\bar{X}<0.6)$ using the Normal approximation (aka. CLT; $\bar{X}$ is the sample mean).

3. Consider a RIS of size 37 from Normal distribution with the mean of 1.4 and standard deviation of 2.7. Compute the probability that

   (a) $\exp(-\bar{X}) < 0.2$,

   (b) $\ln s < 0.9$,

   (c) $\left(e^{-\bar{X}} < 0.2\right) \cup \left(\ln s < 0.9\right)$,

   (d) $|\bar{X} - 1.4| > \frac{s}{5}$,

   where $\bar{X}$ and s are the sample mean and sample standard deviation, respectively.

4. Consider a RIS of size 9 from gamma(2,3) distribution. Find the expected value and standard deviation of

   (a) $4\bar{X} - 7$, where $\bar{X}$ is the sample mean,

   (b) the sample median $\tilde{X}$.

   Compute

   (c) $\Pr(X_{(2)} < 3)$,

   (d) $\Pr(X_{(8)} - X_{(2)} > 7)$,

   where $X_{(2)}$ and $X_{(8)}$ are the 2^{nd} and 8^{th} order statistics, respectively.

5. Consider a distribution with the following pdf

$$f(x) = \frac{x^{-1+1/\theta}}{\theta} \qquad \text{when } 0 < x < 1$$

   (0 otherwise), where θ is a positive parameter whose value is not known. Assuming that we use a RIS of size n to estimate its value, find

   (a) the corresponding Rao-Cramer variance,

   (b) a *sufficient statistic* for such an estimation, and its expected value,

   (c) the *maximum likelihood* estimator of θ,

   (d) its expected value and RC efficiency.

6. Customers enter a store at an average rate of 18.3 per hour (the inter-arrival times are independent, exponentially distributed). Compute the

   (a) probability of getting more than 5 arrivals during the next 15 minutes,

   (b) $\Pr\left(T_5 < \frac{T_8}{2}\right)$, where T_5 and T_8 are the times of the fifth and eight arrival from now, respectively (hint: use beta distribution),

   (c) pdf of $U = \exp\left(-18.3 \cdot T_5\right)$ and its support.

7. In a box there are 4 copper marbles valued 12 pennies each, 9 steel marbles worth 2 pennies each and 14 clay marbles of no resale value. Assuming that 8 marbles are randomly selected (without replacement) from this box, find the

   (a) expected (total) value of marbles thus selected and the corresponding standard deviation,

   (b) probability that this total will exceed 24 pennies (hint: use the corresponding PGF),

   (c) correlation coefficient between the *number* of steel marbles and the *number* of clay marbles in this sample.

Note: Please realize that, for some questions, you may earn only a partial mark due to occasional mistakes. In such cases it is very important to provide as much detail about your solution as possible, preferably in the booklet itself. For example, when you figure out that a question requires the use of chi-square distribution with 20 degrees of freedom, spell it out, etc. Secondly: if you don't enter the final answer in your booklet, I will NOT look for the corresponding solution in your Maple!

Further advice:

- when doing double integration, always *plot* the region over which you are integrating first
- plot all of your cdfs to make sure that they start at zero and end at 1
- integrate every pdf you want to use (or present as an answer) over its full support, to make sure you get 1 (to verify its correctness).